# Supplementary Textbook Chapters S2 and S3: Special and General Relativity

- We will consider just a small part of these – which introduce Einstein's Special and General Theories of Relativity

**Young Einstein**

(better than oldster disheveled version)

# Even younger

# Special Relativity

- Einstein's **Special Theory** begins with a proposed axiom – that **the speed of light in a vacuum will be measured to be the same by all observers, regardless of how they move relative to the light beams.**
- **Ordinary objects in your daily life don't behave that way. So…**
- Sounds very anti-intuitive…. But if you're Albert Einstein, it seemed obvious. And, experiments proved he was right.

# A Logical Consequence is…

- The **measured mass** of a moving object increases with its speed, until it becomes infinite at the speed of light itself. Which means it would take an infinite force to further accelerate it beyond this speed, which is impossible. Therefore...
- The speed of light in a vacuum is the ultimate speed limit. No object will be observed to move faster than the speed of light in a vacuum; 299,972 km/sec
- This is such a fundamental constant (combined with the fact that we can measure time intervals more accurately than we can measure distance) that we now invert the historical definitions of a meter and a second so that a meter is defined in terms of the speed of light, *i.e.* 1 kilometer == the distance a photon moves in 1/299,972.458 of a second

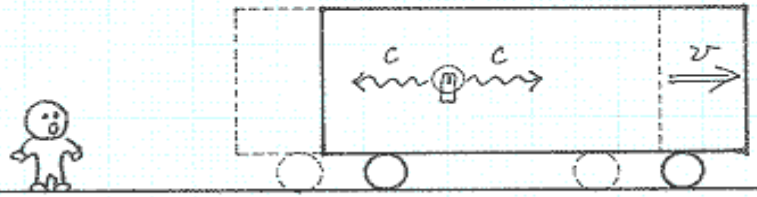
# To Preserve this Fundamental Principle

- Einstein realized the nature of length and time needed to be "relative"; that different observers must measure these differently, else the speed of light could not be a universal constant.
- Hence, moving objects measure shorter in the direction of their motion, and moving clocks tick slower.
- Both of these are necessary so that the speed of light (distance covered per unit of time) would stay constant for all differently moving observers
- The scientific courage of Einstein was in accepting the *relativity* of length, mass, and time, whereas a more conventional mind would simply reject the notion of universal constancy of the speed of light in order to preserve the supposed "universality" of mass, length, and the rate of passage of time

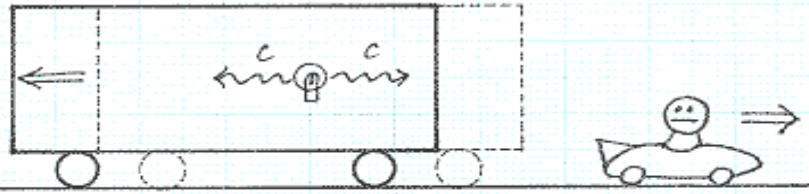
# In Relativity, even the actual order in which events happen in time, depends on the observers' motion

In the frame fixed to the moving train, the beams of light will reach both ends of the train at the same time.

In the frame fixed to the ground, the light reaches the back of the train before it reaches the front.

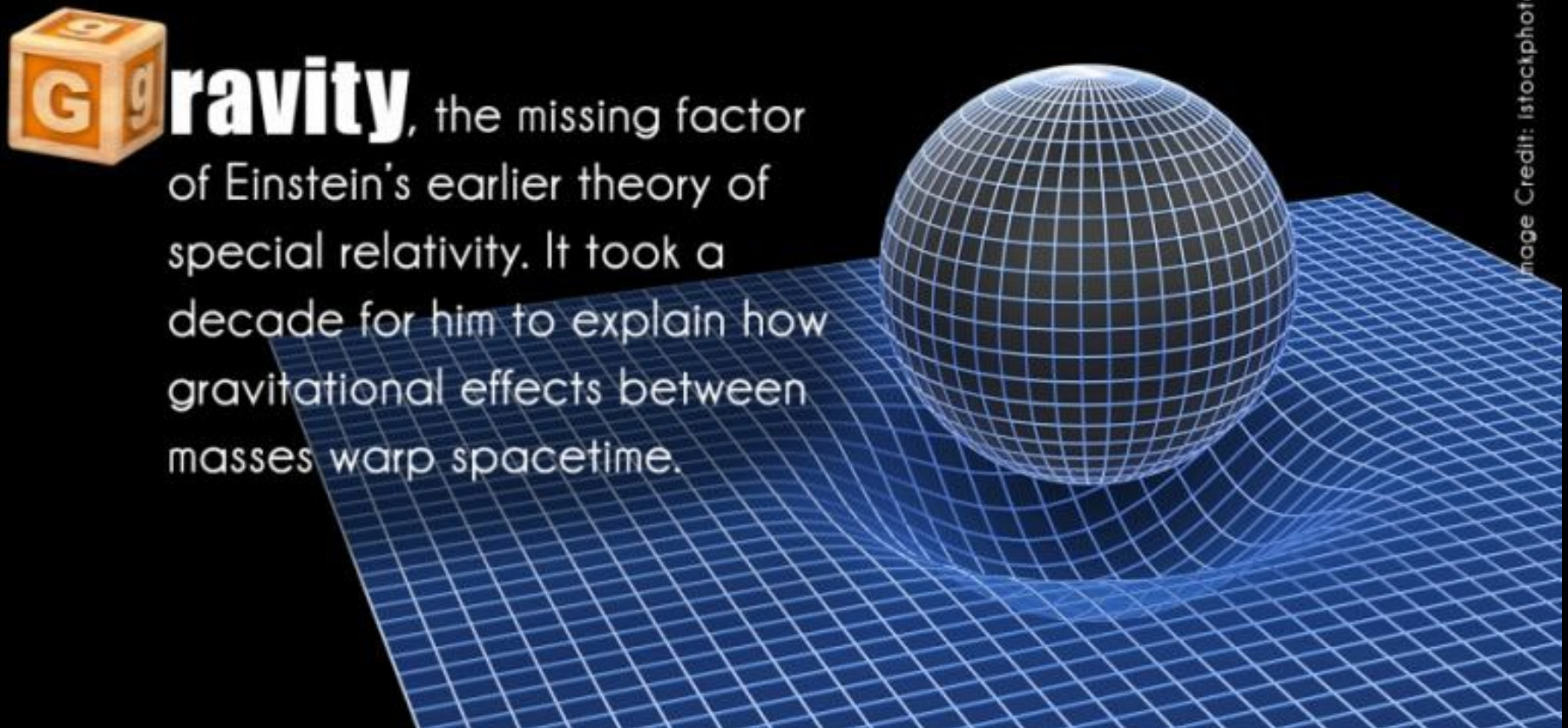



In the frame moving faster than the train, the light reaches the back of the train after it reaches the front.



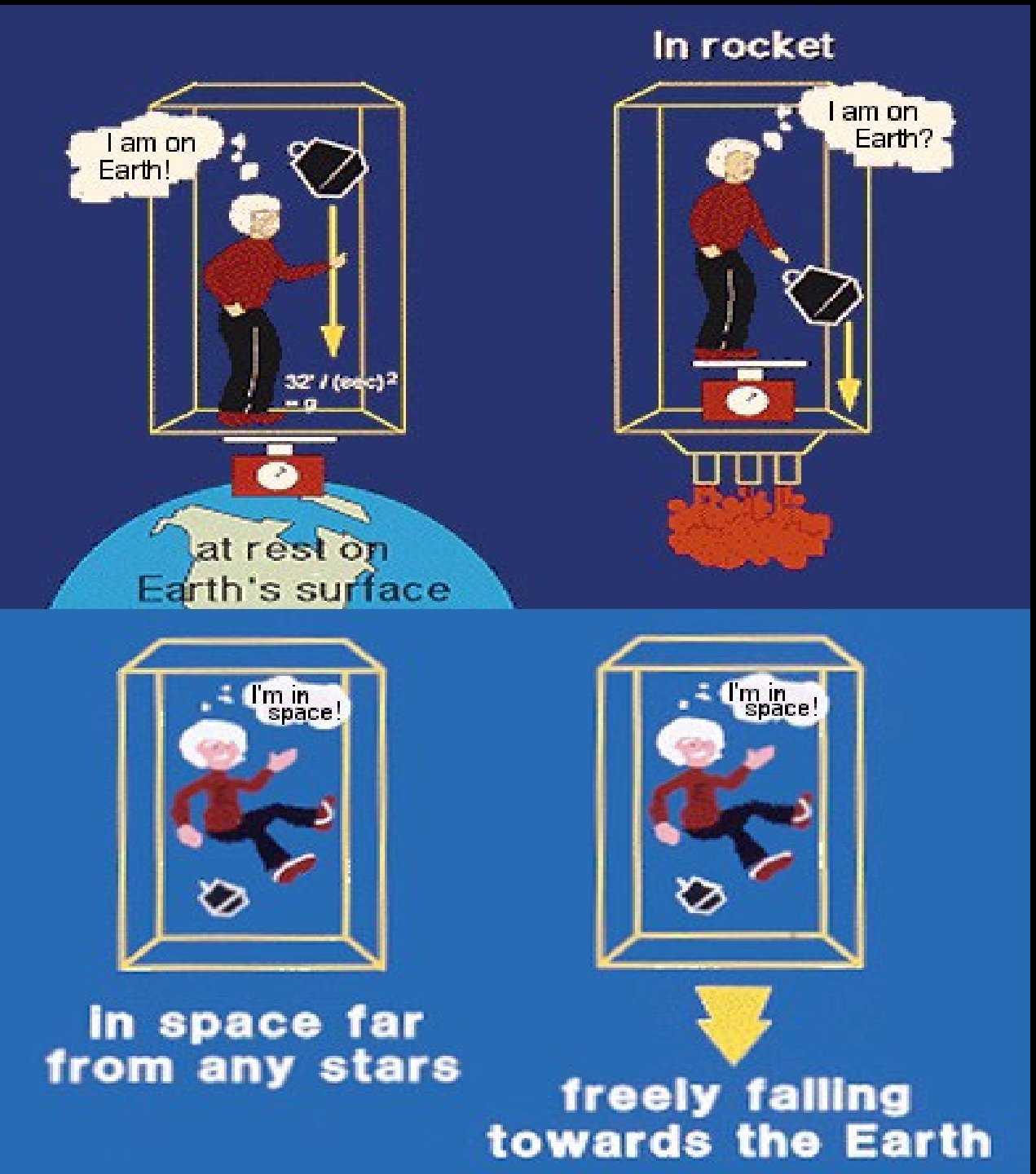
# General Relativity

## Our modern theory of gravity



# General Relativity

- Newton's equation for gravity works pretty well at low speeds and weak gravity such as we're used to - but Einstein discovered by 1915 that the underlying concept of gravity is actually quite different than Newtonian gravity…
- Mass and/or Energy density warps the geometry of the surrounding space. This warped space has a tension, like a bent steel rod carries tension, and this tension is an energy density which itself warps space. Warped space?! But what's there to warp? Actually space has a physical reality to it, even if it has no *mass* to it.
- Einstein realized this and formulated his *General Theory of Relativity* – our modern theory of gravity.
- Took 10 years of really REALLY hard work to develop the mathematics (said Albert himself! - tensor calculus applications)




- Einstein realized there was no difference between being at rest in a gravity field, and being in a uniformly accelerated coordinate frame

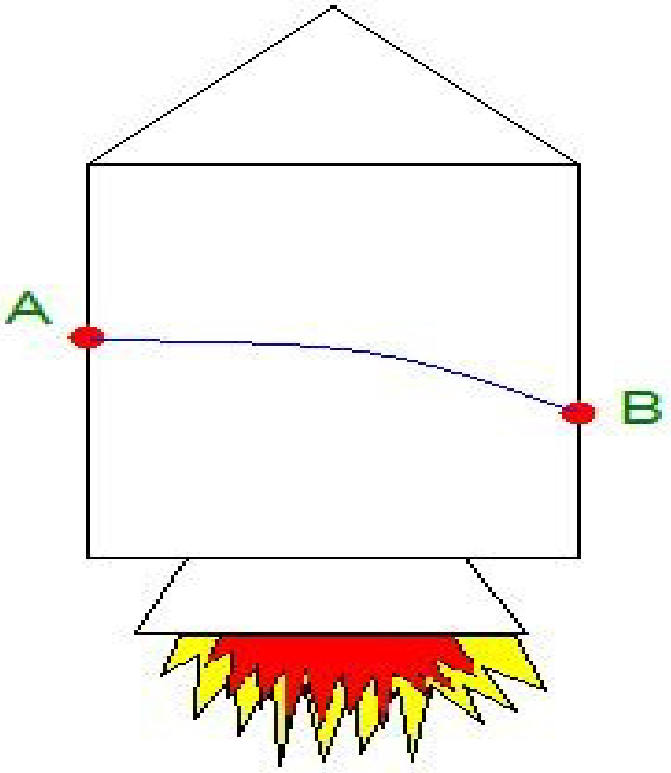
# The Equivalence Principle

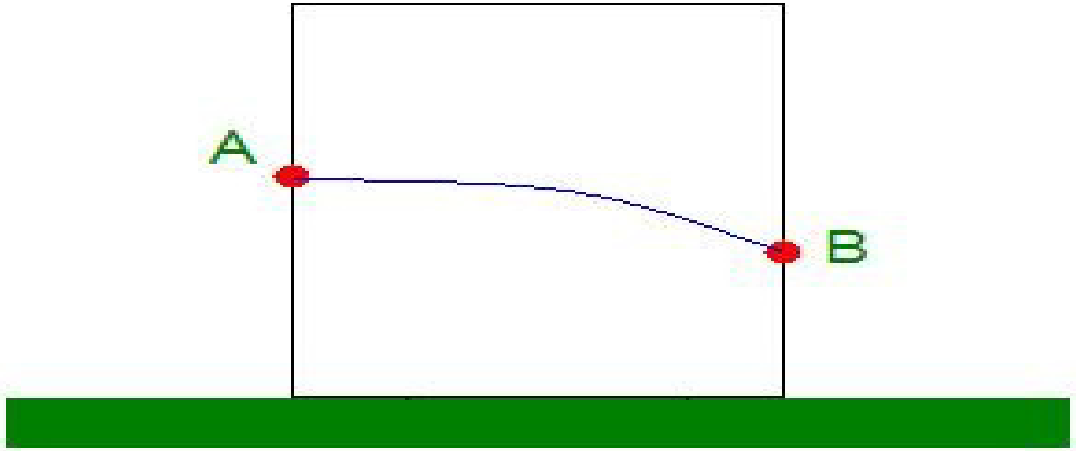
- Einstein noted how the acceleration of gravity could be "removed" if one simply put the falling object into a properly accelerated coordinate frame.
- He took his intuition to the extreme – he felt sure that there could be…

- **NO EXPERIMENT POSSIBLE which could distinguish between being at rest in a gravity field *vs.* being in a suitably accelerated coordinate frame. This is the Equivalence Principle, and predicts the bending of light (next slide)**

- Note that the difference in gravity across the planet (tidal force), means a coordinate frame appropriate to the whole planet must be "rubber" and not just accelerate rigidly, but also stretch appropriately in different places with different gravity.

A laser beam will appear to bend in an accelerating room (left), simply because the room accelerates upward while the beam crosses it - no new physics there. But by the **Equivalence Principle**, a room at rest in a gravity field (right) of equivalent acceleration must show the same bending, despite the fact light, in Newton's physics, was massless and so wasn't expected to bend ($E=mc^2$ says there is a mass equivalent to the energy of light, in Relativity)



# Coordinate Frames

- Einstein became fascinated with coordinate frames. Coordinate systems which could be used to describe natural phenomenon, and how the mathematics got simpler if the right coordinate frame were used. That Nature seems to know about coordinate frames and has Her preferences in given situations. (OK, that's a little anthropomorphic for some people, just so you know).
- The point is, Einstein had an intuition that the messy movings of things under gravity would look far simpler if we instead thought of them as following simpler paths in a curved space. A curved coordinate frame. Later, as it became clear that "empty space" wasn't really "empty", the reality of a curved space had more than just a semantic meaning, it seemed.
- An "Inertial Coordinate Frame" is one in which the laws of physics are homogenous (the same at all places) and isotropic (no direction is preferred over any other, in the laws). Then laws are as mathematically simple as possible
- An inertial frame experiences no a**cceleration.**

# General Relativity is Expressed in Einstein's Field Equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- R = Ricci curvature tensor
- g = metric tensor
- T = stress-energy tensor
- G = Newtonian gravitational constant
- Λ = the Cosmological Constant:

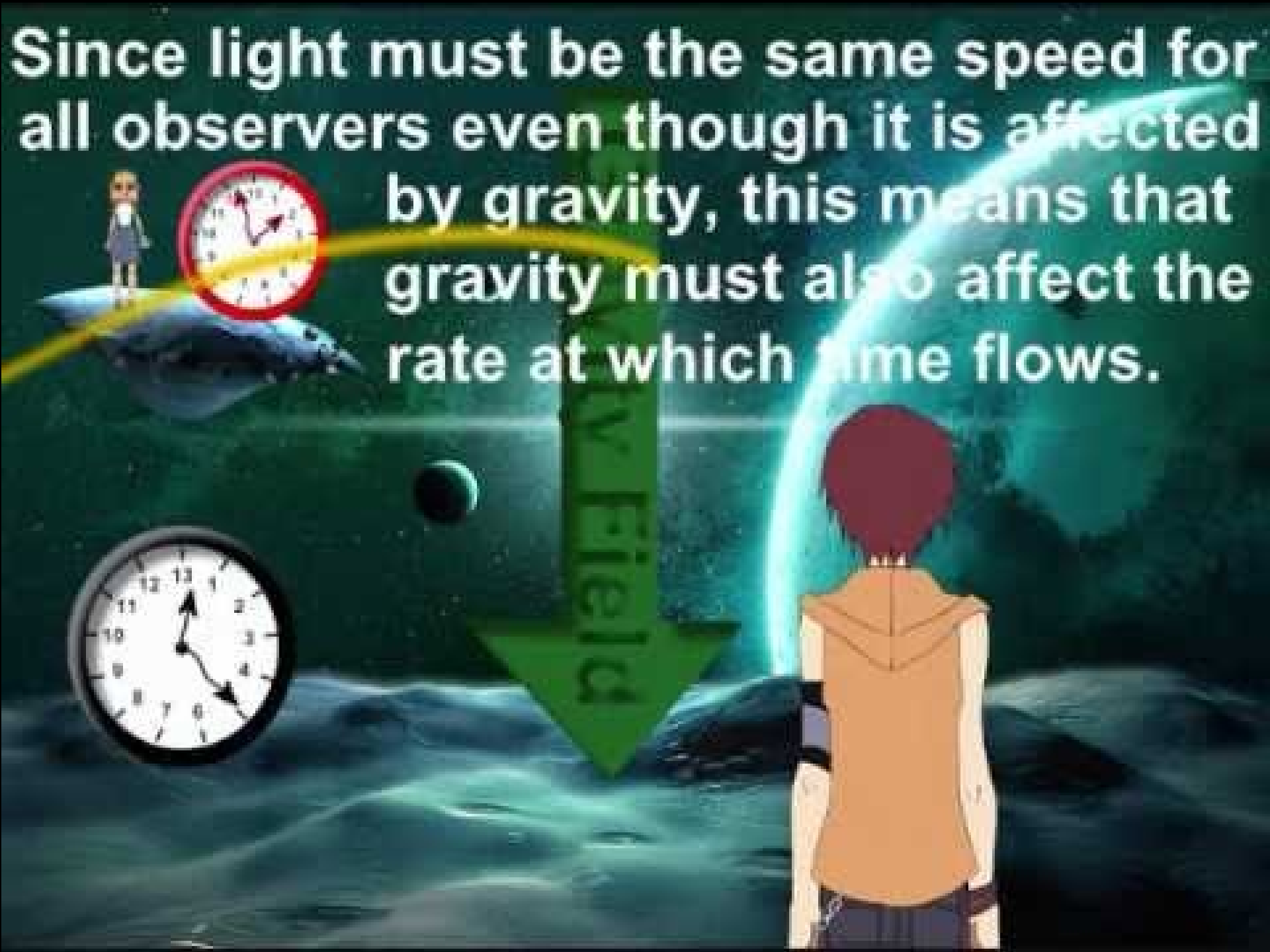
# Even Better When Written on Old Industrial Age Remnants!

# Λ = the Cosmological Constant:

- We'll encounter the Cosmological Constant again, since, surprisingly, it is not zero, but instead about $10^{-52}$ $m^{-2}$
- More precisely, it's measured to be **$0.69(3H^2/(8\pi G)$** where **H** is the Hubble parameter = 67.8 km/s-Mpc, and error bars of about 1.5%
- For now, don't stress on knowing about Λ. When we get to cosmology we'll learn more (and THAT's when you'll need to remember just a bit about it for exam purposes)

# Curved Space!

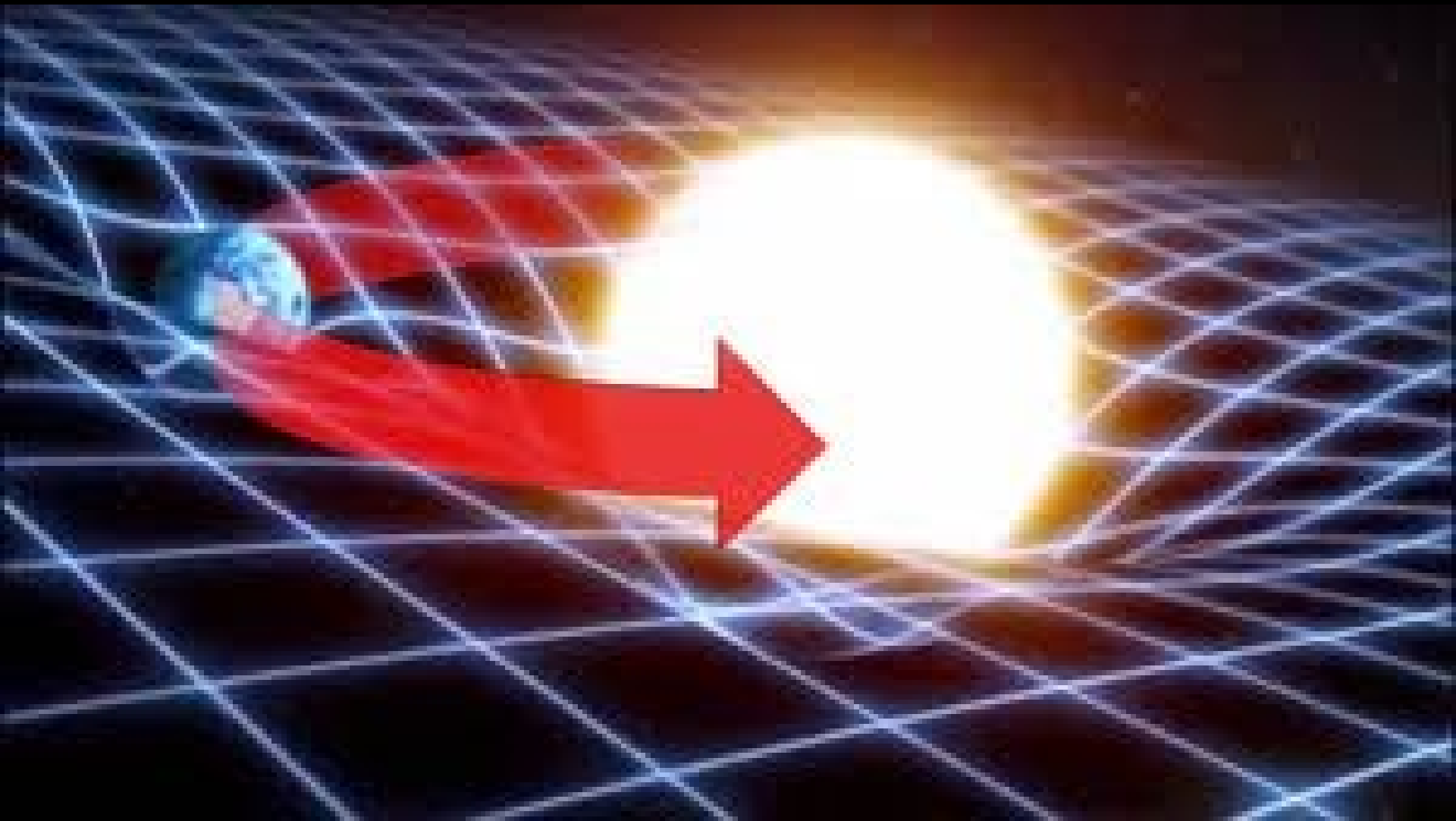
- To model changing gravitational fields over large areas, Einstein realized that the coordinate frames that are “natural” are not made of straight lines, but curved, and “rubber”-like.
- SpaceTime is “curved” by gravity, and gravity is caused by mass, and…
- Special Relativity also showed that $E=mc^2$, and therefore energy has its own equivalent “mass” and will also curve space.
-


Since light must be the same speed for all observers even though it is affected by gravity, this means that gravity must also affect the rate at which time flows.
Gravity Field

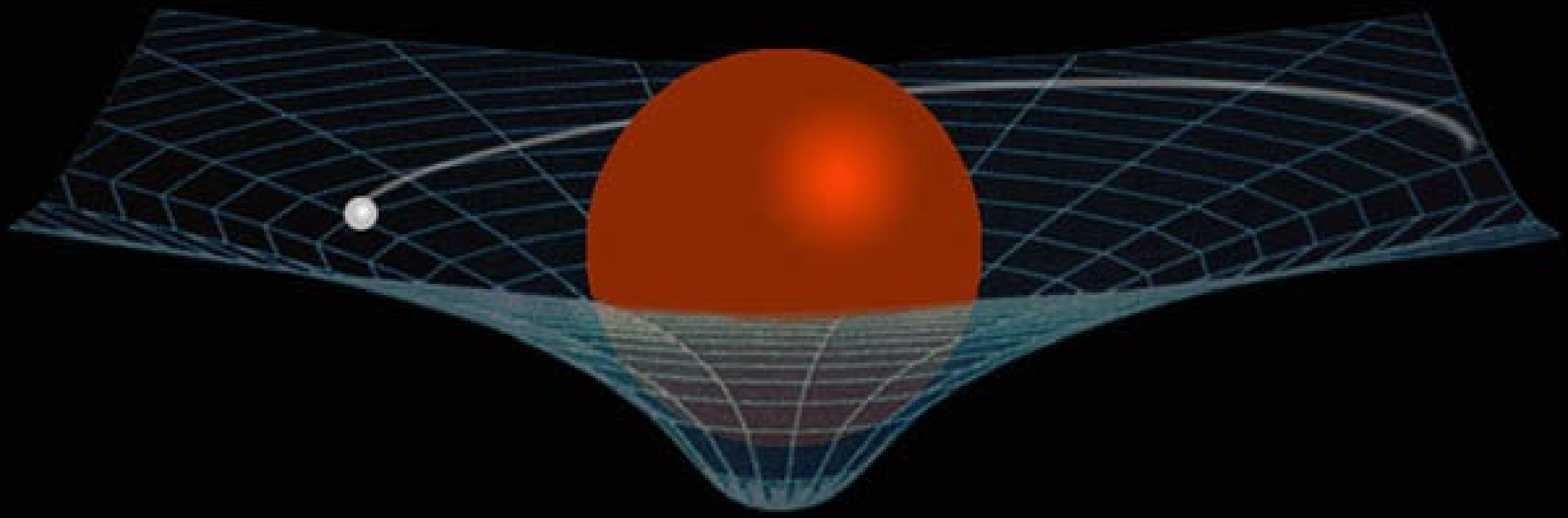
# Here's a YouTube on understanding GR

- It's called …
- Einstein's Field Equations for Beginners!
- Hey, it's only 2 hrs long, and you have lots of free time, so have at it!
- (and, don't believe the title)

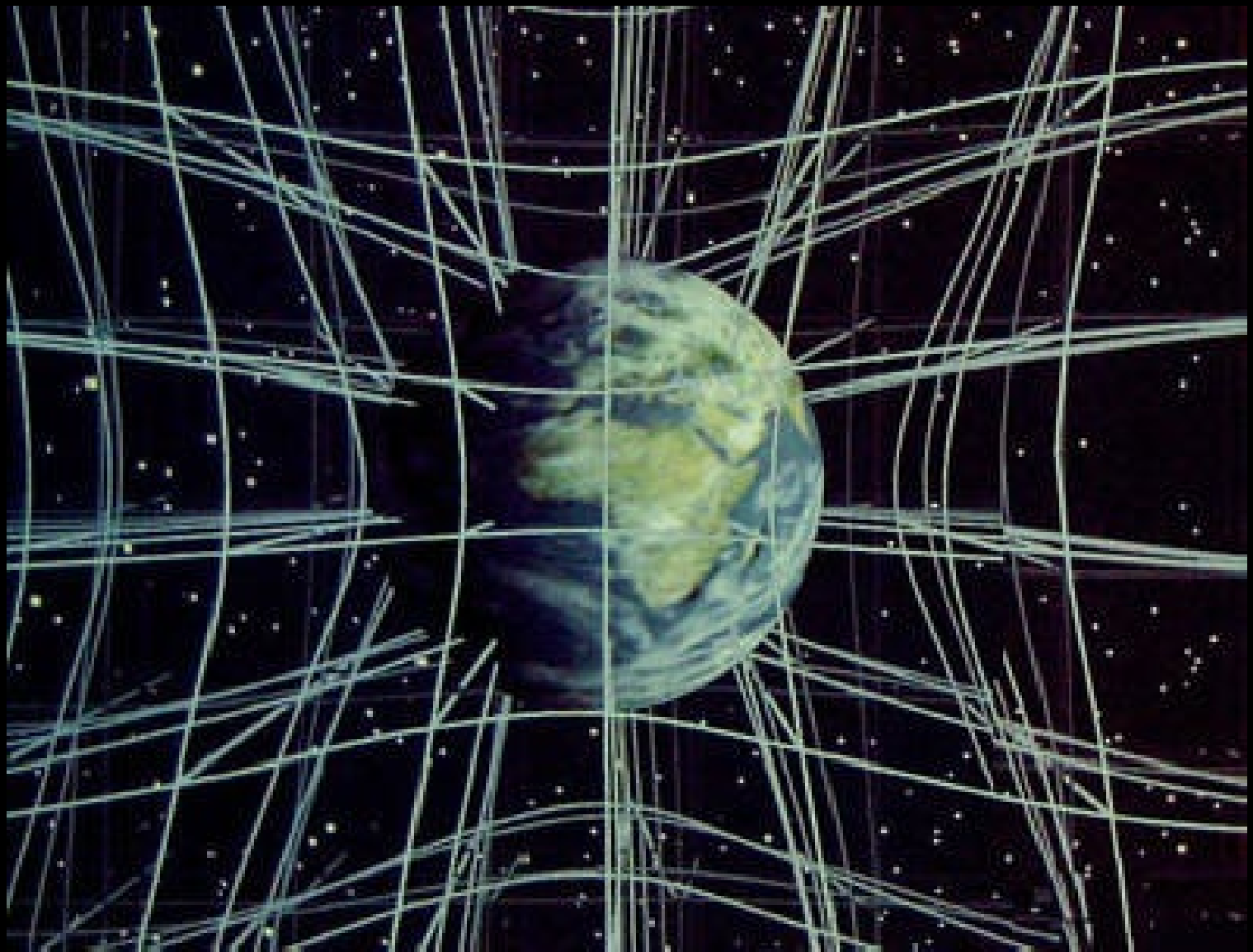
# Orbits, Einstein-style



# An Object Follows an Unforced Path Through a Curved Space, and hence, it Curves



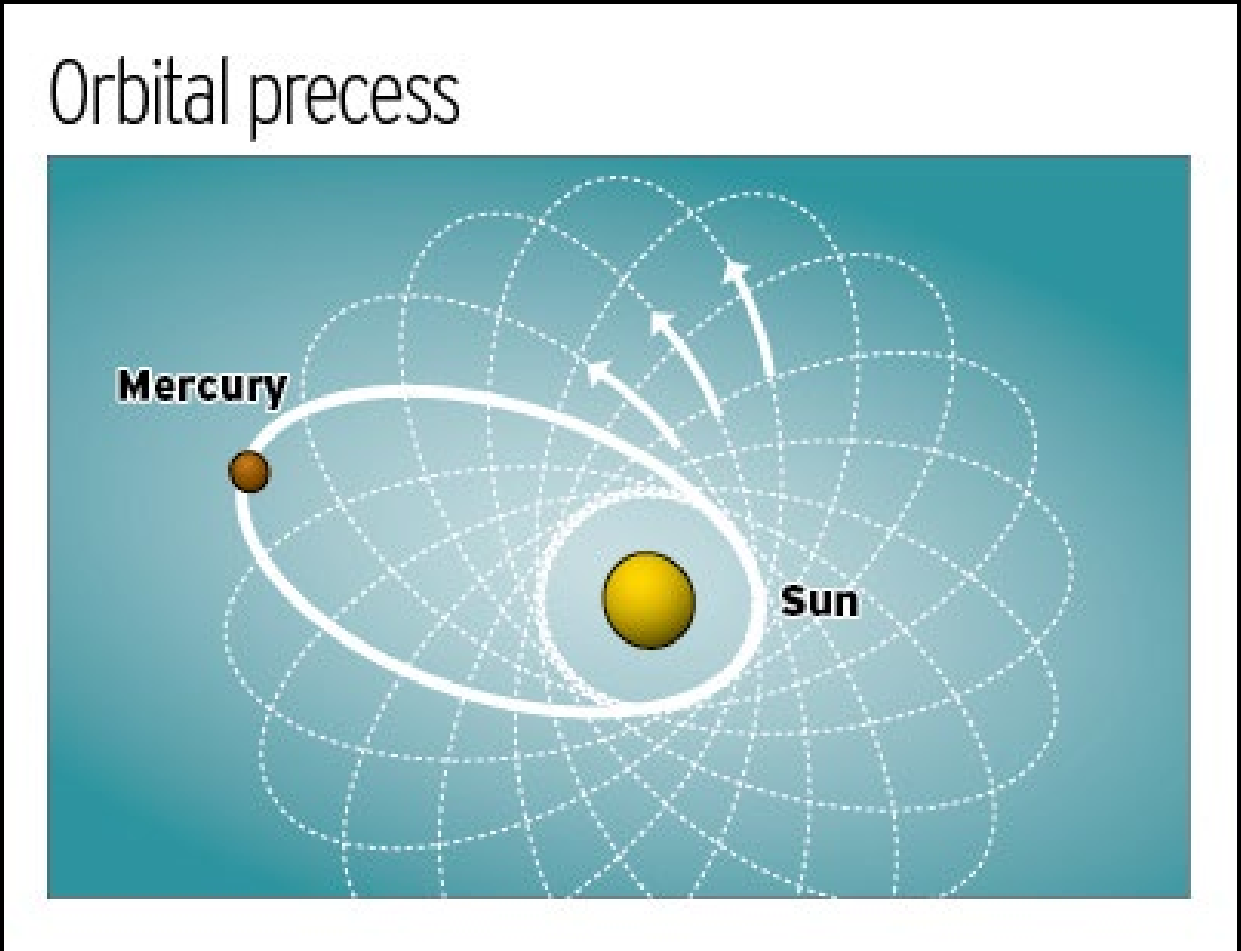
# In 3-D



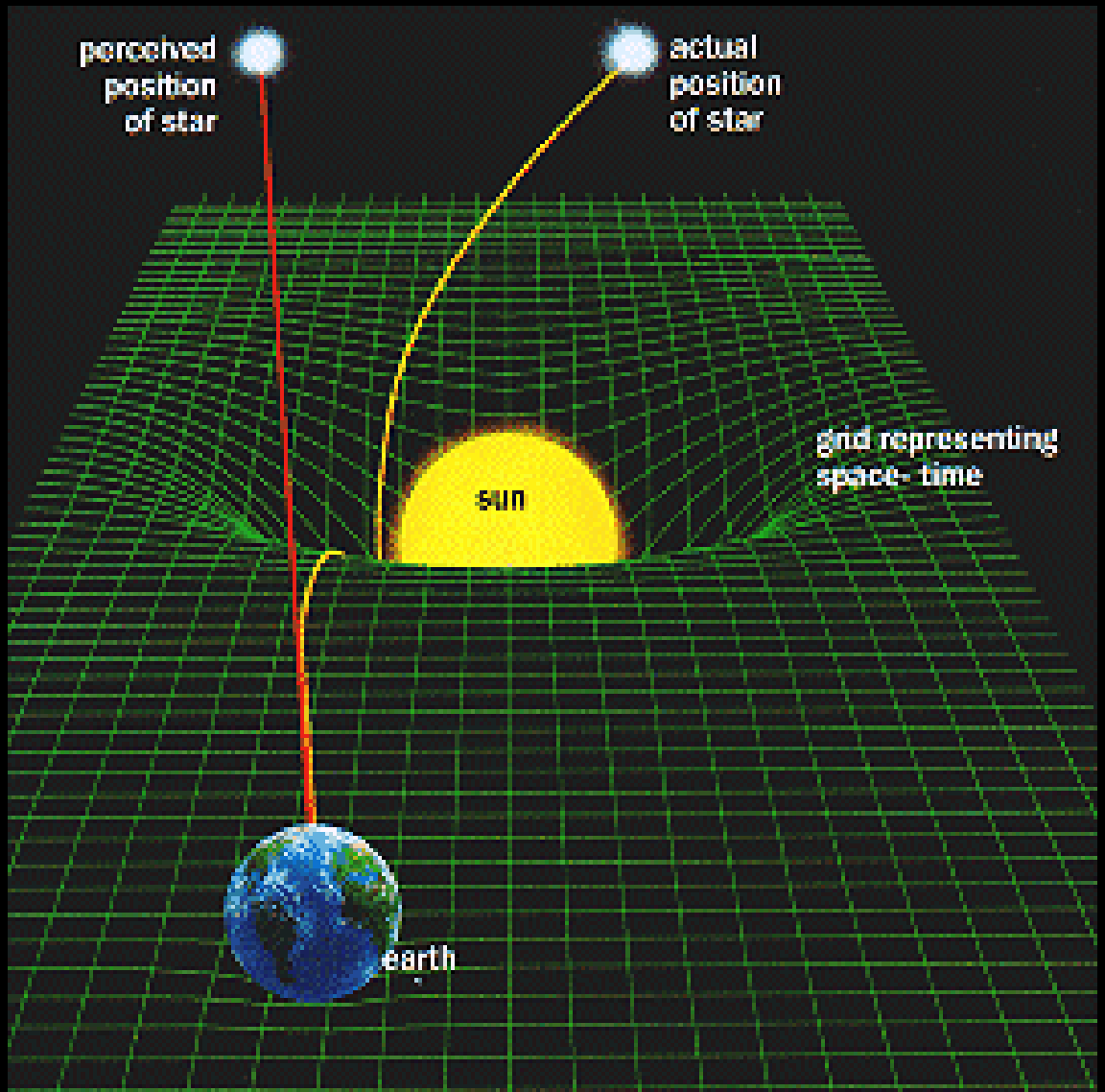
# Putting General Relativity (GR) to the Test

- These realizations allowed Einstein to deduce that there were testable predictions to this radical new understanding of gravity
- This is the hallmark of a true Scientist! Testable predictions which were specific, and uniquely not predicted by Newtonian gravity
- Because he gave testable predictions, his radical new ideas were not dismissed as the rantings of a lowly patent clerk disconnected from Reality.
- Now go read the typical grandiose flowery claims of “New Age” people, who smug’ly provide no objective tests, and note the contrast!

The additional 43”/century precession of Mercury’s orbit was already seen and known by 1916, but unexplained until Einstein showed it was a prediction of GR. It was not, as once hypothesized, due to a non-existent more inner planet named Vulcan (!)

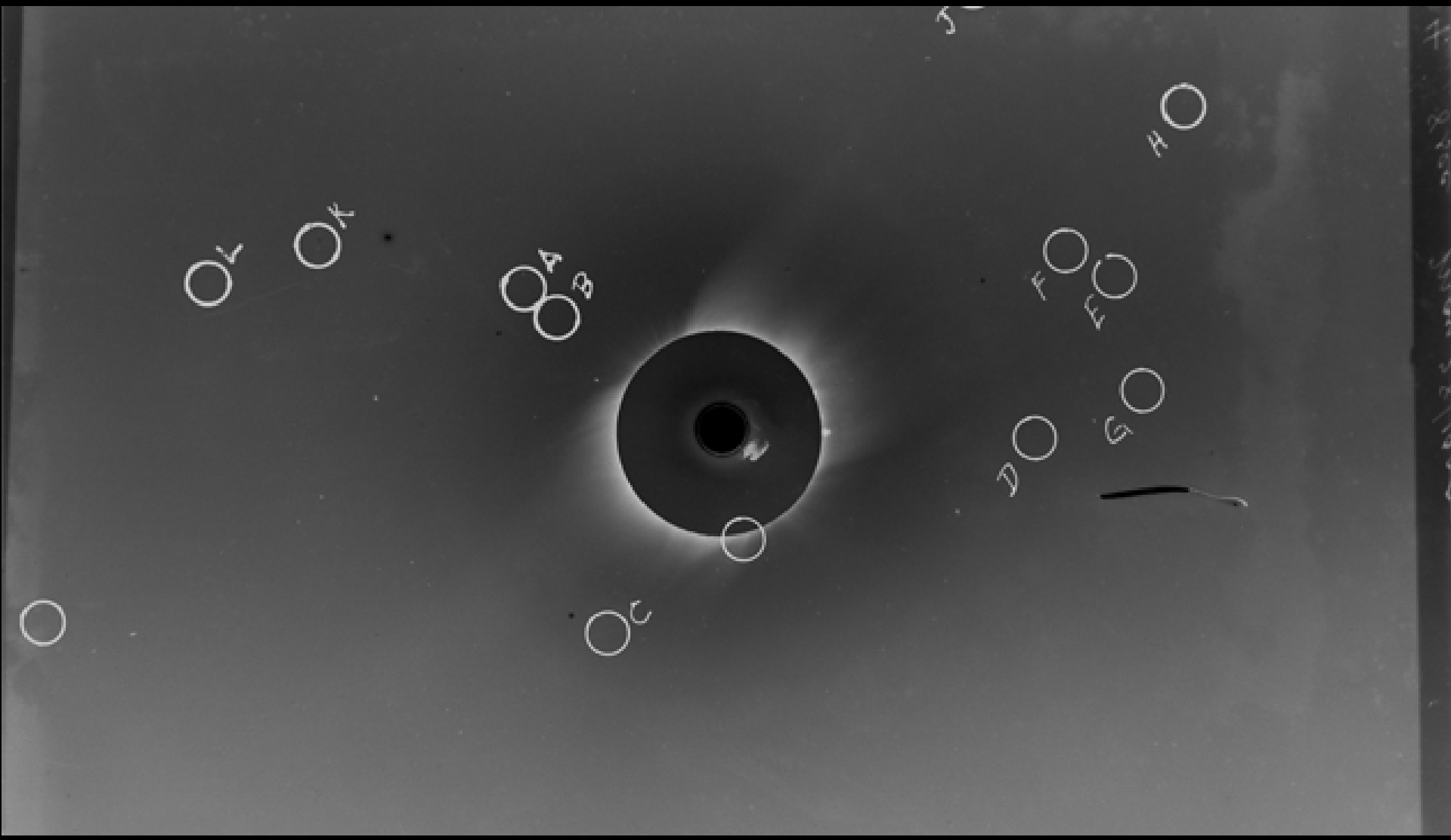


**Light, too, will bend as it moves through space curved by nearby mass. Note how the star appears farther from the sun than it actually is, by the bending**

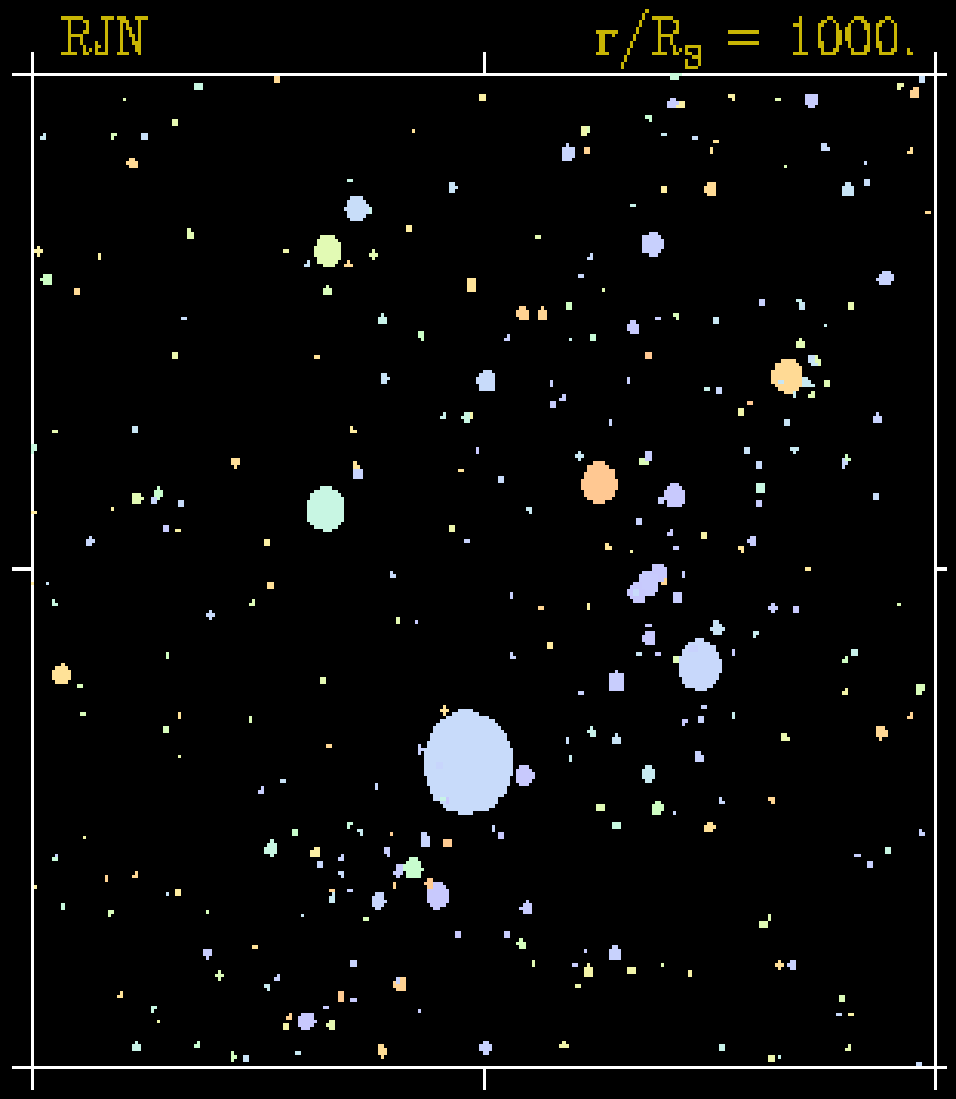


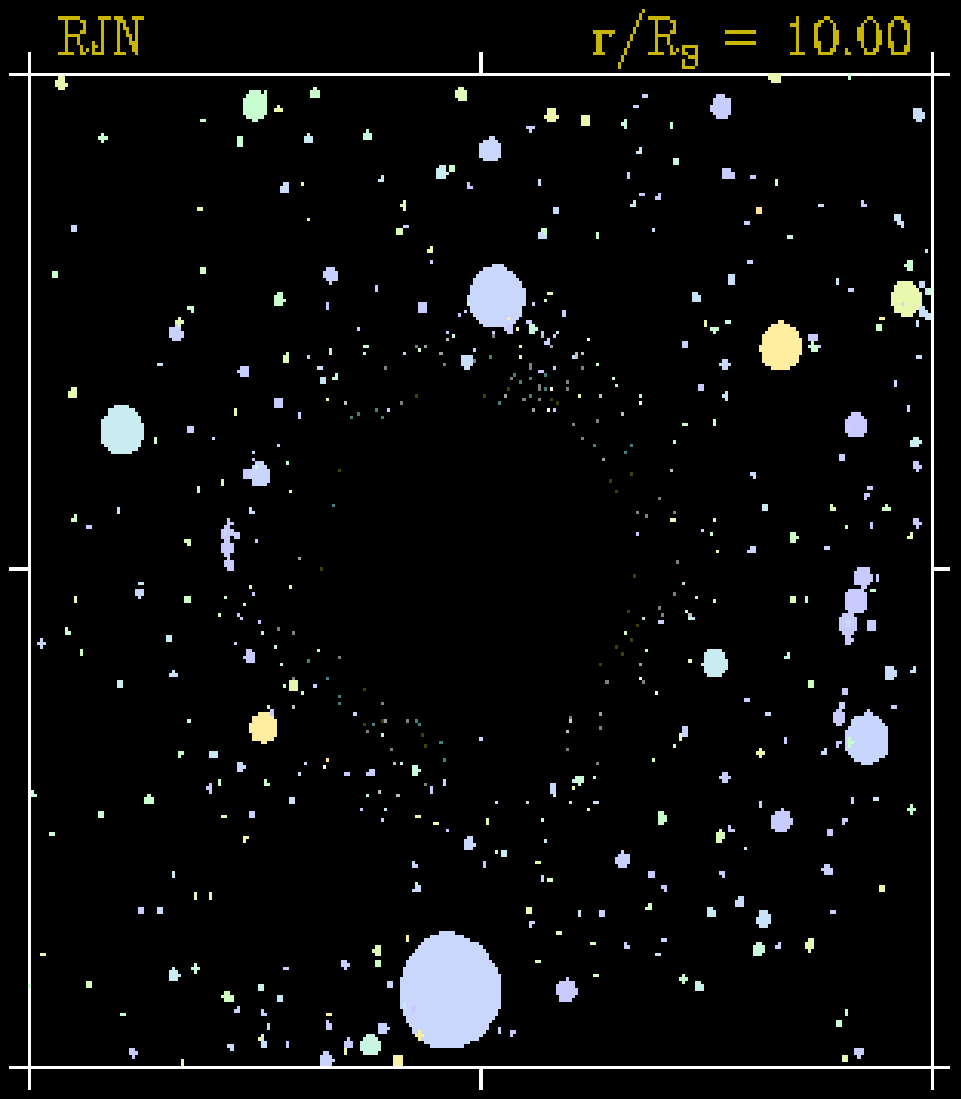
# The Bending of Starlight Confirmed at the 1919 and 1922 Solar Eclipses

# Glass Photographic Plate of 1922 Eclipse with Stars Circled and Measured



# Black Holes hugely bend nearby space. You'd "see" one coming! (which is good)

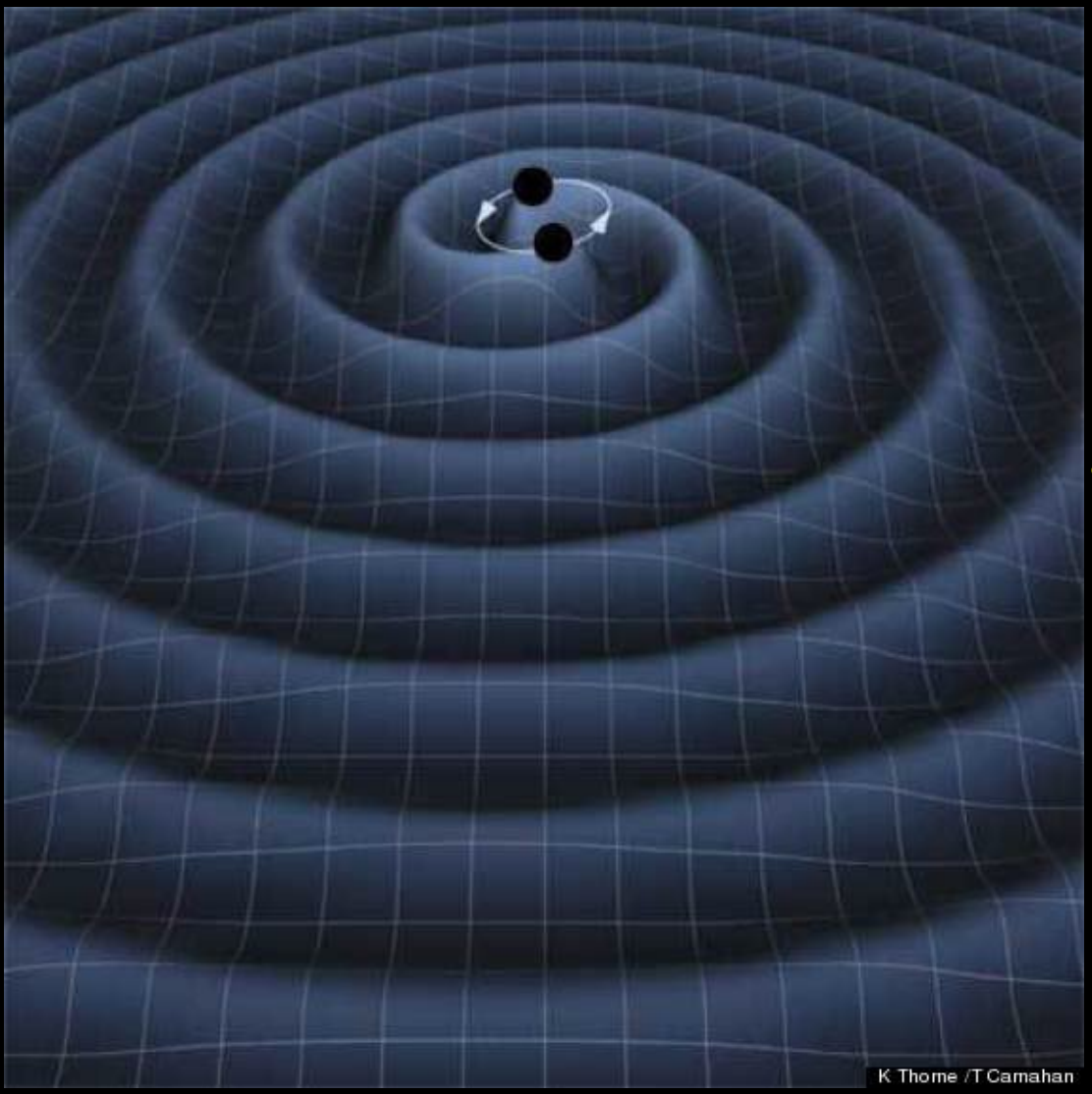


# Later came… new tests

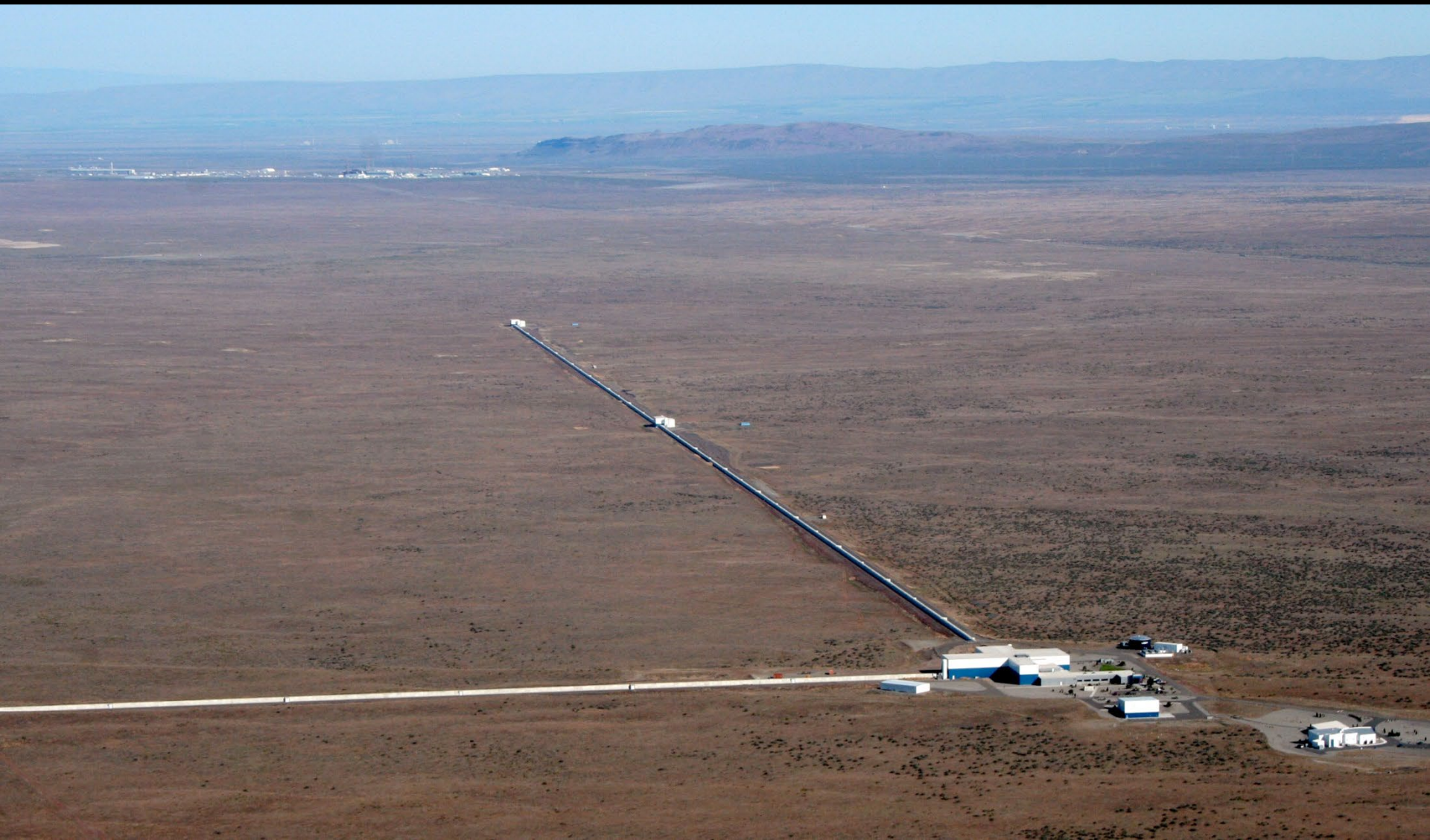
- Black holes in galaxy cores, now too in globular clusters, and binary stars - discovered
- Gravity Probe B and “frame dragging” confirmed
- Gravitational Waves (from the Binary Pulsar) indirectly detected at proper level
- All passed with flying colors.
- However, GR is a “classical” theory – it does not take account of the quantum nature of the very tiny. We still do not have a verified quantum theory which includes gravity.
- One theory getting a lot of attention for many years now is String Theory, in which the fundamental physical units are loops of energy called “string” rather than 0-dimensional elementary particles. This has the benefit of unifying gravity with the rest of quantum fields. However, it makes no testable predictions and also requires 11 dimensions to space/time, *vs.* the 4 actually observed. Some are excited by it because of its beauty, but others are not convinced it’s all that beautiful and await something more testable and obvious in its beauty.
- An interesting extrapolation of Einsteinian conceptions of physics is the ideas of physicist Lee Smolin and colleagues, who consider fundamental physical ideas as relational, rather than involving absolute space and time. The mathematics is extremely difficult and unfortunately his ideas are not yet testable, but for the philosophically and math-minded, worth reading (e.g. here)
- Physics isn’t “done” yet. We’re still working at it…

An “Einstein Ring”, as light from a background galaxy is focused into a ring by the orange foreground elliptical galaxy

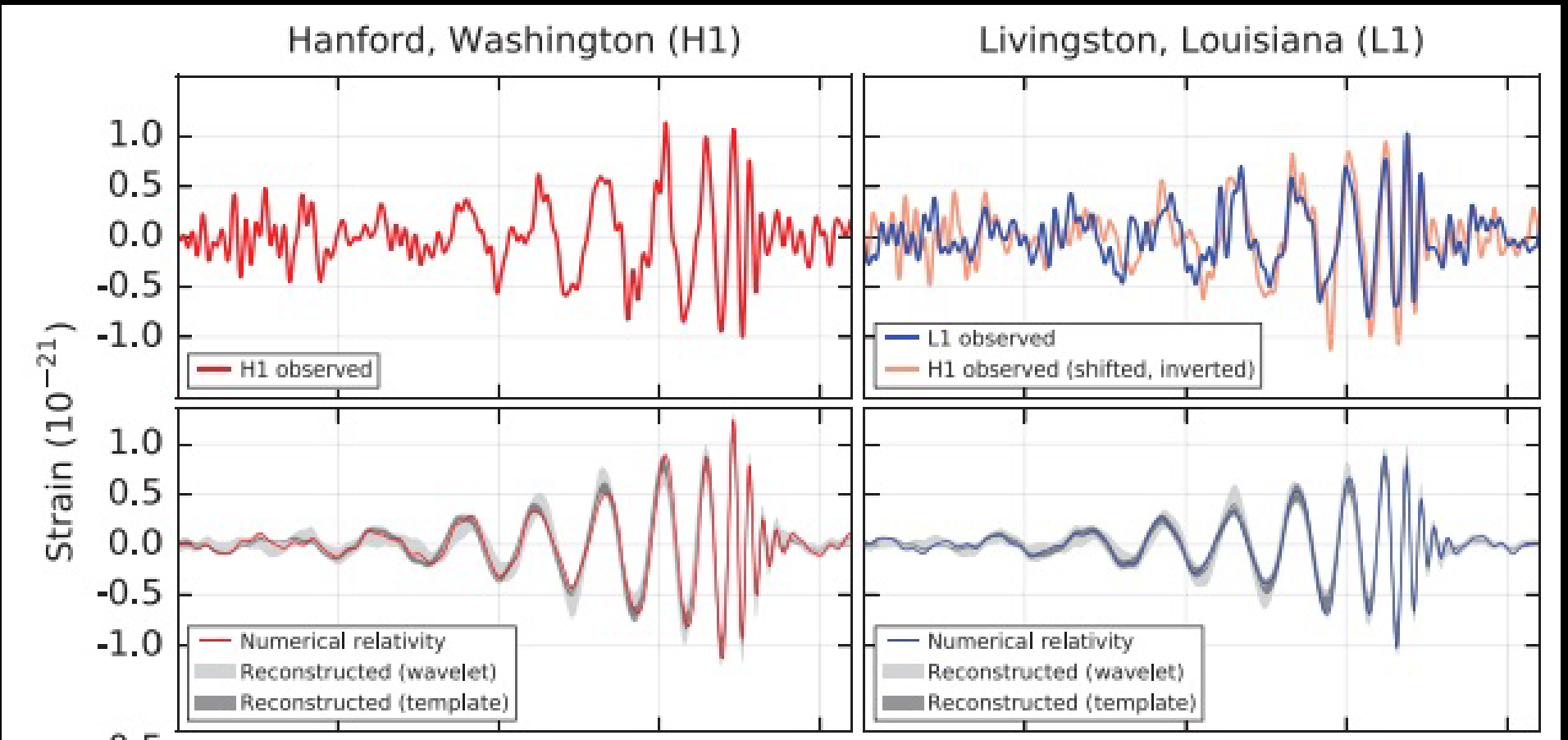



Just as accelerating a charge will create waves in the electromagnetic field, accelerating a mass will create waves in the gravitational field. But since gravity as a force is ~$10^{-37}$ times weaker than EM, the waves are VERY hard to detect

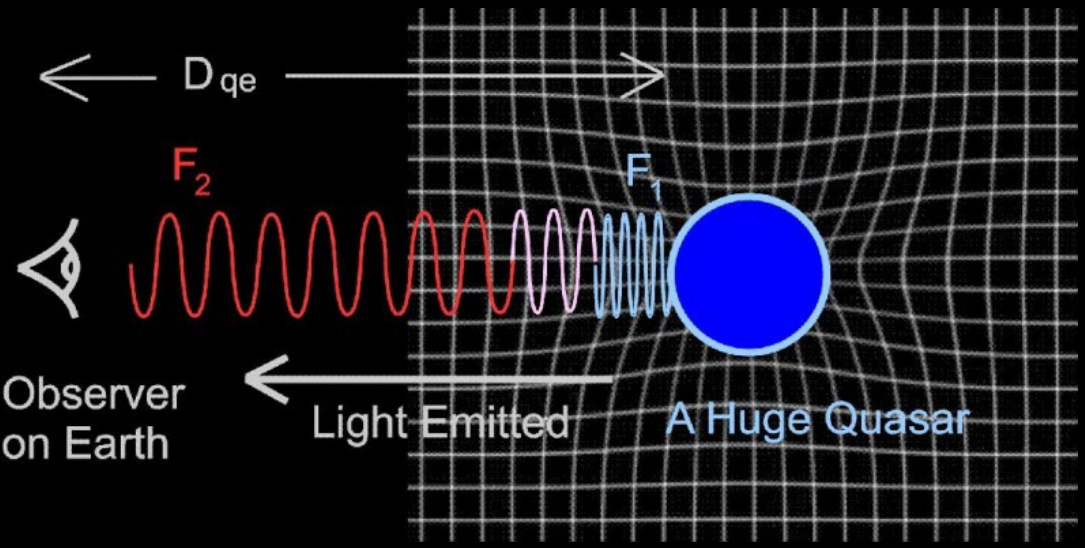
LIGO is two identical interferometers thousands of miles apart using beam splitters to detect the slight contraction along one arm and expansion along the other arm as a gravity wave passes through



**Finally in 2016, we succeeded in direct detection of gravity waves**. LIGO – the Laser Interferometry Gravitational Observatory observed waves produced by the in-spiraling merger of 2 black holes 10's of times more massive than the sun. To put the y-axis numbers in perspective, the 4km long arms expanded and contracted by only 1/250$^{th}$ of the diameter of a proton – and we detected it (!)



Another Prediction of Relativity, is that light must lose energy climbing out of a gravity field. This energy loss shows it self by the frequency and wavelength of the photons as observed by someone beyond that gravity field. **This is called "Gravitational Redshift"** – a new kind of redshift different than the Doppler Effect. This is what makes black holes BLACK! Photons are infinitely redshifted, losing all their energy trying to escape.



# S2: Key Ideas – Relativity

- Special Relativity follows from: All observers measure the speed of light in a vacuum as being exactly the same.
- For this to remain true, Einstein realized measured size and mass the pace of clocks depended on the relative velocity between observed and observer
- Moving masses are more massive.
- Length of moving objects contract along direction of motion
- $E=mc^2$ Energy has a mass equivalent
- General Relativity = our modern theory of gravity, follows from… The Equivalence Principle: The effects of gravity are exactly equivalent to what would be experienced in an accelerated coordinate frame.
- Presence of mass (or energy) warps the geometry of space so that objects moving through that space follow curved paths.
- Slower speed-> more curve. Faster speed->less curve. Speed of light -> very little curve, so we didn't notice till technology advanced
- Gravitational Redshift: Light loses energy and redshifted climbing out of a gravity field. Observationally confirmed