Galaxy Group Analysis - A Robust Discriminator Between Cosmological Models: Cold + Hot Dark Matter and CDM Confront CfA1

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Abstract

We present techniques for identifying and analyzing galaxy groups for the purpose of testing cosmological models. We apply these methods to high-resolution particle-mesh (PM) N-body simulations of structure formation in three $\Omega = 1$ cosmological models: Cold plus Hot Dark Matter (CHDM) with $\Omega_{\rm cold} = 0.6$, $\Omega_\nu = 0.3$, and $\Omega_{\rm baryon} = 0.1$ at b=1.5 (COBE normalization) and two CDM models: b=1.5 and b=1.0 (COBE normalization). Groups are identified with the adaptive friends-of-friends algorithm of Nolthenius (1993). Our most important conclusions are: (1) Properties of groups are a powerful and robust discriminator between Gaussian cosmological models whose Ω_ν differ. We show that our preferred group statistics are robust against several methods for assigning luminosity to dark matter halos, for merging CfA1 data, and for breaking up massive dark matter halos to correct for the overmerger problem. We comment on encouraging new results showing little sensitivity to PM code force resolution. (2) When allowance is made for the higher than typical large scale power present in the CfA1 data, CHDM at our $\Omega_\nu = 0.30$ produces slightly too many groups and too high a fraction of galaxies in groups, while the fraction grouped in CDM is far too low. A slightly lower Ω_ν would appear to produce excellent agreement with all measures, save one: for all simulations, median group sizes are up to a factor of 1.7 larger than equivalently selected CfA1 groups. This appears to be due to residual resolution limitations, but may also reflect a real shortcoming of these cosmological models. (3) The standard group M/L method gives $\Omega \simeq 0.08$ for CfA1 ($V_5 = 350$), and applied to our $\Omega = 1$ simulations gives $\Omega \simeq 0.12$ for CHDM ($V_5 = 350$) and $\Omega \simeq 0.35$ for CDM ($V_5 = 600$) We show quantitatively how three different effects conspire to produce this large discrepancy, and conclude that low observed Ω's need not argue for a low Ω universe. When overmergers are broken up, the median virial-to-DM mass M_{vir}/M_{DM} of 3D selected groups is ~ 1 for all simulations. Groups with $M_{DM} > 10^{14}\ M_\odot$ appear virialized in all simulations. We measure global velocity biases b_v similar to previous studies. Within 3D-selected groups, CHDM and CDM b=1.5 show a stronger bias of $b_v = 0.7 - 0.8$, while CDM b=1.0 shows group b_v's $\simeq 1$.

Subject headings: cosmology: theory — dark matter — large scale structure of the universe — galaxies: formation — galaxies: clustering

1. Introduction

The simplest viable scenario of cosmic evolution begins with an $\Omega = 1$ expansion seeded from inflation with Gaussian primordial fluctuations with a Harrison-Zel'dovich scale invariant spectrum. These fluctuations later collapse via gravitational instability to form structure from galaxies on up. This picture has been remarkably resilient in the face of steadily mounting observations (Gorski, *et al.* 1994, Dekel 1994). The nature of the dark matter will govern how structure forms, and thus may, in principle, be recoverable from the statistics of such structure (if not from direct detection as well). Evidence of structure on very large scales in the late '80's began a series of efforts which have all but ruled out standard Cold Dark Matter (CDM, e.g. Baugh & Efstathiou 1993). This has motivated the investigation of alternative models with the desirable properties of significant power on large scales and low power on small scales, including Cold + Hot Dark Matter (CHDM). As recently emphasized by Pogosyan & Starobinsky (1994), standard CDM's demise leaves CHDM as the only remaining theory compatible with the simplest and most aesthetic versions of inflation. Even more interesting, recent neutrino oscillation experiments provide preliminary evidence that at least one neutrino species indeed has a cosmologically important mass (Caldwell 1995, Primack *et al.* 1995).

Beginning with Klypin, *et al.* (1993, hereafter KHPR), we explore in this series of papers the consequences of a universe dominated by Cold + Hot Dark Matter and compare it with standard Cold Dark Matter and with observations. The CHDM model with $\Omega_\nu = 0.3$ has already shown good agreement with observations of the galaxy correlation function (Baugh & Efstathiou 1993), the redshift space correlation function and galaxy pairwise velocities (Somerville *et al.* 1995) and bulk velocities (KHPR), the cluster-cluster correlation function (Holtzman & Primack 1993; Klypin & Rhee 1994), the variance and skewness of the Abell/ACO cluster distribution (Plionis, *et al.* 1995), the amplitude of the power spectrum from POTENT reconstruction of the local density field (Seljak & Bertschinger 1994), the QDOT-IRAS Redshift Survey power spectrum (Feldman, Kaiser, & Peacock 1994, Fisher *et al.* 1993), quietness of the local Hubble flow (Schlegel, *et al.* 1994), the x-ray properties of clusters vs. redshift (Bryan, *et al.* 1994), and an initial analysis of the properties of galaxy groups (Nolthenius, Klypin & Primack 1994a; hereafter NKP94). The void probability function (VPF) results are less clear. Ghigna, *et al.* (1994) find that the VPF for standard CDM (our CDM1 and CDM1.5) is in good agreement with the Perseus-Pisces Supercluster (PPS) data while that for CHDM is too high. However, Vogeley *et al.* (1994) find that when faint glaxies are included, even CDM1.5, as well as open $\Omega = 0.4$ CDM and CDM with a cosmological constant produce voids which are too empty compared to CfA2. On the other hand, a new analysis of our "2 neutrino" $\Omega_\nu = 0.2$ CHDM simulation (Borgani, private communication) shows very close agreement with PPS data. Also, while CHDM agrees better with the APM galaxy angular correlation function $\omega(\theta)$ than does standard or tilted CDM, CHDM's $\omega(\theta > 3°)$ is still slightly too low (Yepes, *et al.* 1994). Finally, the Hubble constant H_0 can be no larger than ~ 50 km s^{-1}Mpc^{-1} to avoid problems with overproducing clusters and, as for all $\Omega = 1$ models, staying within cosmic age constraints. Therefore, if the current observational evidence for $H_0 \simeq 80$ km s^{-1}Mpc^{-1} is confirmed, then CHDM in its simplest and most aesthetic form is ruled out. Throughout this paper, we assume $H_0 = 50$ km s^{-1}Mpc^{-1}. Aside from this, the easiest way to rule out CHDM is to find massive collapsed objects at high redshift, as CHDM forms structure significantly later than most competing models. Recent observations of damped Lyα systems (Storrie-Lombardi *et al.* 1995, assuming these can be associated with collapsed objects), is already showing a high enough number density at $z \sim 3 - 4$ to be in conflict with the $\Omega_\nu = 0.30$ used here, although $\Omega_\nu \leq 0.25$ appears to still be allowed (Klypin, *et al.* 1995a). We'll show here that group analysis also points towards a lower Ω_ν.

In this paper, we show how the statistics of galaxy groups can provide a powerful discriminator between cosmological models. We identify and analyze groups from the 100 Mpc 512^3 particle-mesh simulations described in Klypin, Nolthenius, & Primack 1995; KNP). The box size was chosen to give a good statistical sample of galaxy groups. With a mesh size of 195 kpc (perhaps typical of an $\sim L^*$ galaxy dark matter halo), we do not have the resolution to study individual galaxies. However, typical galaxy groups have virial radii of $\sim 1 - 2$ Mpc and appear to be adequately resolved. In NKP94 we presented our methods and an initial analysis of groups in the CHDM and CDM simulations, and in the CfA1 Survey (Davis, *et al.* 1982). This paper provides a more complete analysis, and includes corrections for effects not considered earlier. Group analysis combines the information contained in both the velocity structure and the spatial structure of the galaxy distribution. Dave' *et al.* (1995) and Hellinger *et al.* (1995) are complementary studies using these same simulations to devise statistics on the more purely spatial structure of the galaxy distribution.

It can be validly argued that the properties of real groups depend sensitively on the properties of real galaxies, since galaxies are still the tracers for which we have the best observational statistics, and so the need to realistically identify galaxies may not be easy to circumvent. For example, group virial radii depend strongly on the spatial distribution of individual galaxies. Until more realistic large-scale simulations become feasible, the only way to deal with this problem is to add in by hand properties which are well determined physically but poorly understood or difficult to simulate (e.g. luminosity), and to do so equitably across all competing cosmological models. We have addressed the resolution problem by filtering the observational data to the same resolution as the simulations. Fortunately, tests on our chosen statistics show little sensitivity to the relatively poor spatial resolution and related uncertainties. A more significant problem is that of "overmerging" (Katz & White 1993, Gelb & Bertschinger 1994). Dark matter (DM) halos are extended, soft objects which merge easily. Galaxies undergo dissipative collapse to denser, smaller-cross section objects which merge at perhaps only half the DM merger rate (Evrard, Summers, & Davis 1993, hereafter ESD). We considered several schemes for breaking up our most massive halos. While with reasonable assumptions group rms velocities and fractions grouped appear insensitive to the details of halo breakup, we will show it is nevertheless possible to find breakup prescriptions that do lead to significantly different group properties.

We analyze groups selected from two different versions of our catalogs. In order to gain insight into the properties of our models using all available information, we select groups from the complete 100 Mpc simulation boxes using full three dimensional (3D) information ("box groups"). To make meaningful, direct comparisons with the CfA1 data, we first make magnitude limited "sky catalogs" in redshift space from each simulation, and then identify groups using identical criteria. The CfA1 catalog used here was extracted from the 1987 version of ZCAT.

The construction of the 512^3 grid of cells within the 100 Mpc box and the simulation calculation methods are described in KNP. Briefly, we use the "cloud-in-cell" approximation (Hockney & Eastwood 1981; eq. 5-21) for both the density assignment and force interpolation. The Poisson equation was solved using finite differences. A 7-point "crest" template was used to discretize the Laplacian operator, leading to Green's functions analogous to the 2D case (ibid; eq. 5-205). The resulting system of linear equations was then solved by the Fast Fourier Transform technique. In NKP94, we presented a robust and discriminatory statistic, rms group velocity $v_{\rm gr}$ (the rms velocity of all galaxies within the group, i.e. the conventional "velocity dispersion") vs. the fraction of catalog galaxies in groups $f_{\rm gr}$, and compared CHDM with CDM and with the CfA1 Survey data. Here, we present a more complete description of the construction of improved galaxy redshift catalogs, including corrections for some effects not considered earlier, analyze the properties of 3D-selected groups from the full box, and present other comparisons between these catalogs and observations. An accompanying computer visualization video sequence (Brodbeck, *et al.* 1995; BHNPK) compares visually the differences between the simulations, sky catalogs, and the real universe. CDM structures are rather puffy filaments, with clusters at the intersections. By contrast, CHDM shows remarkably delicate filaments (reminiscent, in an imaginative sort of way, of a well prepared egg flower soup). Blurring in redshift space reduces the visual impression of these differences, yet their effect remains powerful on the statistics studied here.

Grouping in redshift space is done using the adaptive algorithm of Nolthenius (1993; N93), although tests with the original algorithm of Nolthenius and White (1987; NW) gave essentially the same results but with slightly lower group velocity dispersions. The other principal grouping method applied to redshift catalogs is the heirarchical scheme (e.g. Tully 1987). Heirarchical methods, however, produce a more severe ceiling on individual group velocity dispersions which would likely reduce our ability to discriminate between models. Recently, Frederic (1995a,b) has claimed that the NW grouping algorithm seriously underestimates group velocity dispersions and hence M/L ratios when compared with simulation groups. However, this study fails to emphasize the importance of the underlying cosmology to setting the grouping algorithm's appropriate link in redshift. Using only a low bias $\Omega = 1$ standard CDM simulation, as Frederic did, will indeed show that the NW redshift link normalization of $V_5 = 350$ km s^{-1} is too low (as already shown in NKP94) due to the less concentrated structure and high pairwise velocities in this model. The fault lies not in the algorithm, but in the assumption that this version of CDM is an appropriate calibrating cosmology for determining the merits of grouping algorithms used on real observations. One must first show that carefully constructed simulation data sets give properties closely similar to those of the corresponding observations *over a broad range* of grouping linkages. Only then can the simulation be used to optimize grouping algorithm parameters. This is the approach taken here, and in NW.

2. Construction of the Galaxy Halo Catalogs

A detailed description of the simulation calculations is given in NKP94 and KNP. Briefly, we use a particle mesh code on a 512^3 grid, with 256^3 cold and 2×256^3 hot particles. The cold particle mass is $2.9 \times 10^9\ M_\odot$ and $4.1 \times 10^9\ M_\odot$ for CHDM and CDM, respectively. As before, we refer to the CDM b=1 (COBE normalization, see Smoot, *et al.* 1992) and b=1.5 simulations as CDM1 and CDM1.5, respectively. The two CHDM simulations, CHDM1 and CHDM2, are both at b=1.5 (COBE normalization), and differ only in their initial conditions. The CHDM$_1$, CDM1, and CDM1.5 simulations all began with the same random number set describing the amplitudes of the initial waves perturbing the particles. It was later found that this random number set had abnormally high power on large scales. The power spectrum was a factor of ~ 2 higher than typical on scales comparable to the box size. The probability of this occuring in any given realization was estimated at $\sim 10\%$ (KNP). A second CHDM simulation, CHDM$_2$, began with a much more typical spectrum. This fact will be important later in interpreting the comparisons with CfA1 data. As it turns out, there is good evidence that CfA1 has unusual large scale power as well. The value of beginning each competing model with the same random perturbation set is that it guarantees the same large scale structures will emerge in each, so that differences between simulations will solely be due to differences in the underlying physics of the evolution and not to cosmic variance.

A galaxy halo is defined as a mesh cell with a sufficiently high dark particle mass overdensity $(\delta\rho/\rho)_{cut}$ at the end of the simulation (z=0). We quantify the mass of a halo by the mass contained within a cell (or sometimes 3x3x3 cell) boundary. This is obviously crude. The boundaries are arbitrary and no attempt is made (nor is it possible) to include only the gravitationally bound particles as our spatial and force resolution is too poor to justify such refinements. Nevertheless, as long as galaxy luminosity monotonically rises with 1-cell mass, our results are insensitive to how mass is quantified. We decided against using a popular alternative method of identifying halos, the DENMAX prescription (Gelb 1992), which merges all particles within a local density enhancement by moving them up the local density gradient. Besides being computationally intensive, we were concerned that this procedure would merge too much surrounding mass to be properly associated with the visible galaxy. This may be a problem for cosmologies in which present day structure is still relatively young, such as CHDM.

When our purpose is to optimally delineate structure we keep all cells above $(\delta\rho/\rho)_{cut} = 30$, giving N=29151 halos in CHDM1, N=37,164 in CDM1, N=45,592 in CDM1.5, and N=29,795 in CHDM$_2$. When constructing magnitude limited redshift catalogs and attempting to match CfA1 galaxy number densities, experiments showed that $(\delta\rho/\rho)_{cut} = 80 - 150$ was best, cutting galaxy totals by about 70%. Note that this is slightly less than the $(\delta\rho/\rho)_{cut} = 170$ corresponding to virialization, (e.g. Kaiser 1986), and seems reasonable if one assumes virialization should actually apply to a denser core of material closer to the visible galaxy. Since each cell is 1/512 of the 100 Mpc box, or 195 kpc , a cell would most properly correspond to a dark matter halo surrounding a typical L^* galaxy. Two measures of the mass of such cells were calculated: the dark particle mass contained within the cell, and the dark particle mass contained within a 3^3 cell cube (3-cell) centered on the cell of interest.

How sensitive are these results to our limited force resolution? While the cell size of 195 kpc is only 1/5 of the typical size of a group, the force resolution of our simulation may substantially affect the identification of the tracer galaxies. To check this, we have recently run two 100 Mpc box PM simulations of a "2 neutrino" model suggested by the preliminary analysis of the LSND experiment (Caldwell 1995): one at 512^3 and one at 800^3 resolution, with identical parameters and initial conditions. While the galaxy identifications do differ substantially, the group results do not. The 800^3 box, when halos are identified on a 512^3 resolution, gives redshift space group statistics which are virtually identical to those of the 512^3 box. At full 800^3 resolution (which included some additional refinements in our procedure), $f_{\rm gr}$ rises and $v_{\rm gr}$ drops slightly, both by about 1 standard deviation for all grouping linkages. However, making the approriate re-adjustment to the resolution of the CfA1 Survey (see §5) takes observed results in the same direction, and much of this small shift is removed. In either case, there are no changes in our conclusions, and our resolution appears adequate for this analysis. We save discussion of these simulations for a later paper.

3. Breaking Up Massive Halos

One uncertainty in our earlier results is the possibility that the overmerging of dark matter galaxy halos may have significantly lowered the rms velocities of galaxies within groups, and perhaps also lowered the total fraction of galaxies in groups. The dominant galaxy will tend to sit near the group center and have low center-of-mass velocity. By contrast, baryonic dissipation causes earlier collapse into several smaller but higher velocity galaxies (Katz & White 1993) whose merging rate is slower (ESD). If, as has been argued by some, the overmerging seen in numerical simulations is mostly due to poor mass resolution, our results should be relatively secure. Our mass resolution is quite good; an L^* galaxy has about 100 cold particles and twice as many hot particles. Assuming, however, the more likely probability that the lack of dissipation is indeed the dominant source of overmerging, we now ask above what halo mass M_{bu} does it become important? Katz & White (1993) find that some individual halos could be as massive as $1.5 \times 10^{14}\ M_\odot$. However, using reasonable M/L ratios, cooling efficiencies, and star formation rates, they argue most halos above $M_{bu} = 5 \times 10^{13}\ M_\odot$ should be broken up, and that M_{bu} could be as low as $2.8 \times 10^{12}\ M_\odot$. Gelb (1992) finds too many halos above $V_{circ} = (\frac{GM}{r})^{1/2} = 350$ km s^{-1} which, for r corresponding to our cell size, is equivalent to $3.4 \times 10^{12}\ M_\odot$. ESD also address this issue, and with better mass resolution they find a lower M_{bu} limit of $7 \times 10^{11}\ M_\odot$. Our most massive CHDM halo has 3-cell mass of $5 \times 10^{13}\ M_\odot$ (or a 1-cell mass of $5 \times 10^{12}\ M_\odot$). To date, ESD is the best published work available on galaxy formation in groups and clusters in a dark matter background. Their Figure 12(a) provides a rough relation between dark matter halo mass and the number of galaxy-like objects ("globs" in their nomenclature) within the halo in standard CDM. While this figure applies to z=1, their most recent results show the number of globs stays fairly constant to z=0, at least in standard CDM (Evrard, private communication). They define the halo's mass as the mass within the sphere containing an average overdensity corresponding to virialization; $\delta\rho/\rho = 170$. Assuming unevolving halo size from z=1 to the present, the cosmological expansion factor of 2 then gives a corresponding $\delta\rho/\rho = 2^3 \times 170 = 1360$ for M_{bu} in our z=0 simulations. We found the radius r_{eff} and mass M_{eff} of this sphere by determining the radii of spheres encompassing the volume of our 1-cell and 3-cell masses, and assuming density fell linearly from the 1-cell sphere radius to the 3-cell sphere radius. The maximum radius allowed to enclose a single breakup candidate correponded to the sphere enclosing a 2^3-cell volume, or halfway to the nearest allowable halo. This was done to avoid double counting some of the exterior mass. We then broke up halos whose mass M_{eff} within a sphere of average overdensity $\delta\rho/\rho = 1360$ was greater than $7 \times 10^{11}\ M_\odot$. The result of this procedure was $N_{om} = 138$ (CHDM$_1$), 157 (CHDM$_2$), 571 (CDM1.5), and 658 (CDM1) halos identified as overmergers and suitable for breaking up into fragments. If mass is roughly proportional to light, observed galaxies suggest that fragments should be assigned Schechter-distributed masses. Dissipational hydrodynamic codes also produce gaseous galaxy-like objects with Schechter distributed masses (Evrard, private communication, Frenk, *et al.* 1995; FEWS). We therefore constrained the fragments to follow a Schechter distribution with the characteristic mass $M^* = \langle M/L \rangle L^* = 4.3 \times 10^{11}\ M_\odot$ (CHDM) or $1.08 \times 10^{12}\ M_\odot$ (CDM). L^* is $4.3 \times 10^{10}\ L_\odot$ from the CfA1 and $\langle M/L \rangle$ is the median M/L of the simulation halos: 10 for CHDM and 25 for CDM. The faint end slope was set to the merged CfA1's $\alpha = -1.26$ (see §6). Each overmerger was replaced with fragments of total mass M, where M was the virialized overdensity mass M_{eff} calculated above, minus the mass expected to be in fragments below the overdensity limit $(\delta\rho/\rho)_{cut}$ selected for the simulation (and thus too faint to see): 80 for both CHDM and 156 for both CDM simulations (see §6). For all simulations, $16 - 19\%$ of the integrated Schechter function is contained in fragments below $(\delta\rho/\rho)_{cut}$. For each overmerger, the brightest fragment was given a mass $M_{bf} = (M_{bu} M_{eff})^{1/2}$ (Evrard, private communication) and placed at the original overmerger's position. Remaining masses were then randomly selected from a Schechter distribution and randomly placed into any halo which could accept it without overfilling. This continued until the next random fragment mass was too large to be added to any halo. At this point, all halos were $\sim 99\%$ full of fragments. Each set of fragments for a given halo was then ordered by mass and then sequentially placed as close as possible to the parent halo cell while still enforcing the 2-cell closest neighbor resolution limit. Thus the most massive fragments were placed closest to the original DM center. Each fragment was then given a randomly oriented, random Gaussian velocity with dispersion equal to the rms velocity V^{om}_{neigh} of all halos within 1 Mpc of the overmerged halo (or, in the few cases this did not include at least 4 halos, out to the 4-th nearest halo). The actual velocity distribution in such circumstances is poorly known, but simulations to date suggest Gaussian is approximately correct (Evrard, private communication). The median values $\langle V^{om}_{neigh} \rangle_{med}$ of these neighborhood rms velocities were $\sim$ 300 km s^{-1} for all models, but individual overmergers ranged as high as $\sim$ 1200 km s^{-1} (CHDM) to $\sim$ 2000 km s^{-1} (CDM1). The most massive fragment was left at the original overmerger's position. Observations indicate that the brightest (assumed the most massive) galaxies in groups and poor clusters are moving at a center-of-mass velocity only $\sim$ 0.25 that of their lower luminosity neighbors (Bird 1994). We therefore multiplied the brightest fragment's velocity by 0.25. The fragments of a given overmerger do not, at this point, satisfy conservation of momentum. In the frame of the parent overmerger, the net momentum of the N fragments with masses m_i is $\sum_i^N m_i \vec{v_i} = \vec{p}$. To enforce momentum conservation we correct each fragment velocity by adding a velocity differential $\vec{\delta v} = -\vec{p}(\sum_i^N m_i)^{-1}$. Finally, the masses were rescaled by M_{1cell}/M_{eff} so that they followed the 1-cell convention used for all other cataloged halos. 3-cell masses were found in a similar way.

The simulation halos we analyze below have all been subjected to this, our preferred breakup scheme. We also experimented with other breakup schemes. These are described in detail below, and summarized in Table 1.

The first alternative scheme, hereafter "Method 1", made use of ESD's Figure 12(a), giving the number of galaxies per DM halo vs. DM halo mass. M_{bu} was again $7 \times 10^{11}\ M_\odot$, but this time the halo mass was simply assumed to be the 1-cell mass; generally a bit lower than the mass inside a sphere of average overdensity 1360. We broke each overmerger candidate into ESD's nominal number of fragments, giving them essentially equal masses (when ordered by mass, each fragment was an arbitrary 10% more massive than the previous fragment). Velocity assignments were as before, except rather than using all neighbors inside a distance of 1 Mpc, we used the 10 nearest neighbors, whatever their distance (typically out to a distance of ~ 2 Mpc ; as large as a medium sized group). The most massive fragment's velocity was not reduced. Placements again enforced the 2-cell nearest neighbor limit, but this time without regard to putting the most massive fragment closest to the DM center. The largest overmerger in any simulation spawned only 7 fragments, and the large majority of overmergers produced only 2 or occasionally 3 fragments. By essentially maximizing the mass of each fragment, this prescription guaranteed (albeit unintentionally) the highest possible number of fragments surviving the magnitude limit and making it into the final sky catalogs (see §6). Since these galaxies survived in close pairs or groups, it also significantly raised the fraction of galaxies in groups.

The second scheme ("Method 2") made the more reasonable assumption of Schechter-distributed masses, randomly selected and assigned with the same M^* parameters as our adopted scheme, but with ESD's "glob" faint end slope of $\alpha = -1.35$. M_{bu} was the same as for our adopted scheme. We did not constrain the brightest fragment's mass or velocity (except by enforcing momentum conservation, as before). Fragments were positioned randomly, regardless of mass, but enforcing the 2-cell limit. By not insuring at least one reasonably massive fragment, the result was a larger number (~ 30 on average) of low mass fragments, relatively fewer of which ultimately survived the magnitude limit for inclusion into the sky catalogs. Velocities were randomly sampled from a Gaussian distribution with dispersion equal to the circular velocity $V_c^{om} = (GM_{eff}/r_{eff})^{1/2}$ associated with the virialization mass and radius described in our final method above.

Our preferred scheme assigns fragments their nearest neighbors' rms velocity V_{neigh}^{om} rather than the circular velocity just described, for several reasons. First, the most appropriate mass and radius to use are unclear. Second, galaxies form early, and, in the dense environments common to these overmergers, it is the larger tidal field of the group which seems likely to eventually determine final fragment velocities. Finally, one of our goals was to evaluate how well the virial theorem measures mass in these systems, and enforcing dynamically determined velocities would bias these results. Figure 1 shows a scatter plot of V_c^{om} vs. V_{neigh}^{om} for each overmerger in the CHDM and CDM simulation boxes. The V_c^{om} and V_{neigh}^{om} distributions differ substantially. However, their median values are very similar $\langle V_c^{om}\rangle_{med} \simeq \langle V_{neigh}^{om}\rangle_{med}$, as shown in Table 2. Circular velocities are slightly higher than V_{neigh}^{om}; by $\sim 5\%$ for CHDM and by $\sim 15\%$ for CDM. Thus, using circular velocities would likely have raised our final group rms velocities by only a few percent, more so for CDM than for CHDM. (However, it might also be argued that a more appropriate dynamical velocity would be closer to the virial velocity, which would be lower than V_c by $\sim 2^{-1/2}$). Note also that the extended tail of high V_{neigh}^{om} is especially pronounced for the CDM models, and is related to CDM having higher pairwise velocities than CHDM (KHPR).

Another approach ("Method 3") is to leave the simulation halos alone and instead attempt to put real galaxies back into "overmerged" halos by merging them, i.e. taking a luminosity weighted averaged of their positions and redshifts, then combining luminosities. In this case, one wants to merge CfA1 galaxies which are within about $1.5 - 2$ cells (~ 350 kpc) on the sky, and within a velocity separation corresponding to the virial velocity of these massive halos, i.e. about $200 - 300$ km s^{-1}. Since this is typical of the rms velocity of a medium sized group, Method 3 turns out to correspond closely with the merging scheme already done in NKP94 (although our motivation then was actually to correct for spatial resolution, not overmerging). As we will see, the NKP94 results are quite close to the results of our more careful analysis here. We regard Methods 2 and 3, and especially Method 1, as less realistic than our adopted procedure. The assumptions for each of our breakup methods are summarized in Table 1.

Table 1. Summary of Overmerger Breakup Methods

Method	Assumptions
Preferred	$M_{bu} = 7 \times 10^{11}\ M_\odot$, M inside $\delta\rho/\rho > 1360$
	Schechter fragment masses M, $M^* =< M/L >_{sim}\ L^*_{CfA}$, $\alpha = \alpha_{CfA} = -1.26$
	$\vec{v}_{fragment}$ =Gaussian (σ,$\vec{v}^{\dagger}_{om}$), σ=rms of galaxies within 1 Mpc or nearest 4
	Fragments positioned by mass, higher mass closer to center
	Brightest fragment mass $M_{bf} = (M_{bu} M_{eff})^{1/2}$, at overmerger center,
	and velocity $\vec{v}_{bf} = 0.25 \times \vec{v}_{fragment}$
Method 1	$M_{bu} = 7 \times 10^{11}\ M_\odot$, M inside $\delta\rho/\rho > 1360$
	No. fragments from ESD Fig 12a, masses, if ordered, each 90% of previous mass
	$\vec{v}_{fragment}$ =Gaussian (σ,$\vec{v}^{\dagger}_{om}$), σ=rms of 10 nearest galaxies
	Fragment positions not mass-dependent
Method 2	$M_{bu} = 7 \times 10^{11}\ M_\odot$, M inside $\delta\rho/\rho > 1360$
	Schechter fragment masses M, $M^* =< M/L >_{sim}\ L^*_{CfA}$, $\alpha = \alpha_{ESD} = -1.35$
	$\vec{v}_{fragment}$ =random Gaussian (σ,$\vec{v}^{\dagger}_{om}$), $\sigma = (GM_{eff}/reff)^{1/2}$
	Fragment positions not mass-dependent
Method 3	Leave simulation overmerges alone
	Merge CfA1 grouped galaxies within $r_{proj} = 235$ kpc , i.e. NKP94 results

fragment positions enforced 2-cell limit, for all methods
† $\vec{v}_{om}$ = velocity of original overmerger

Figure 2 shows the 1-cell mass distribution of the halos. The no breakup halos follow a power law of slope $d \log N/d \log M = -1.33$. We stress that the 1-cell and 3-cell masses, being defined by arbitrary boundaries, are not particularly physical measures of dark matter halo mass, and in our analysis are used only as stepping stones to luminosity. All breakup methods lead to a fairly sharp cutoff in masses at the upper end. Note that Method 1 makes an especially noticable pile of fragment masses just below the breakup mass limit M_{bu}.

All of these breakup schemes may in fact overestimate the number of fragments. Other purely dissipationless simulations (e.g. Carlberg 1994) find that dense DM cores are suprisingly persistent within virialized clusters, suggesting that overmerging may be less significant than generally believed. Also, if $M^*/10$ is equivalent to a luminosity of $L^*/10$, ESD and Evrard (private communication) find too many fragments by an order of magnitude, when compared to observations. Our preferred procedure produces even more fragments (a median of 8 fragments per overmerger for both CDM simulations and 14 for both CHDM simulations, with a maximum of ~ 140 for all). Our luminosity assignment method (see §6) prevents overpopulation of visible galaxies, but perhaps does not prevent too high a fraction of visible galaxies which are fragments. On the other hand, FEWS find that the number of baryonic "galaxies" within a DM halo is significantly higher when they turn dense baryonic globs of $M > 10^{10}\ M_\odot$ into stars at $z = 0.7$. FEWS conclude that we still do not have robust methods of simulating galaxy formation within DM halos. Our favored statistics, fortunately, turn out to be fairly stable against reasonable breakup assumptions, and quite similar to our no breakup results.

4. Properties of Box Groups

We first describe groups selected using full 3D spatial information from a complete sample of all galaxies above a rather low 1-cell mass cutoff. Our procedure for constructing redshift space "sky catalogs" that can be compared to CfA1 data are described in §6. A group is defined as the galaxies within a bounding surface of constant number overdensity. Our grouping scheme is the standard "friends-of-friends" algorithm. The critical link distance D is found from the N galaxies in the box above the mass cut by $D = D_0(l^3/N)^{1/3}$, where l is the length of the box and D_0 is the link parameter, expressed as the fraction of the mean interparticle spacing. For each galaxy, all neighbors within a distance D are linked. Each of the newly linked neighbors is in turn searched, until no more members are found. We adopted $D_0 = 0.36$ (corresponding to a number overdensity for selected groups of ~ 20), for our comparisons, yielding a fraction grouped of $\sim 70\%$ for CHDM and $\sim 60\%$ for CDM. This overdensity limit best corresponds with the redshift selected groups described here and in earlier work (e.g. Huchra & Geller 1982, Nolthenius & White 1987 (NW), Ramella, *et al.* 1989 (RGH), Nolthenius 1993 (N93)). In constructing groups near the box boundaries, the periodic images of galaxies were also considered during linking. This avoided the problem of clipping groups near the boundaries artificially.

Table 2 shows properties of groups made from the full box. The CHDM $(\delta\rho/\rho)_{cut} = 80$ and CDM $(\delta\rho/\rho)_{cut} = 156$ boxes will later be assigned luminosities and generate observational sky catalogs, as these halo density cuts best match CfA1 galaxy densities (see §6). We refer to these below as our fiducial boxes. To facilitate comparison between CHDM and CDM at the same halo mass density cut, we also include $(\delta\rho/\rho)_{cut} = 80$ CDM box results below. Note that these box groups are made from a sample containing a high fraction of low mass galaxies and groups, unlike the later groups to be made from magnitude limited sky catalogs.

Table 2. Properties of Box Groups

simulation	$CHDM_2$	$CHDM_1$	CDM1.5	CDM1
halo $(\delta\rho/\rho)_{cut}$	80	80	156	156
N^*_{halo}	11305(8585)	10898(8134)	14636(8503)	16050(7247)
N^{**}_{om}	157	138	571	658
V_c^{om} km s^{-1}	309	316	315	322
V_{neigh}^{om} km s^{-1}	283	319	266	291
N_{grps}	641(620)	576(575)	727(680)	619(514)
$\langle M_{vir}/M_{DM}\rangle_{med}$	1.26(1.02)	1.43(1.18)	1.27(1.40)	1.44(1.96)
$f_{\rm gr}$ †	.71(.70)	.73(.71)	.58(.50)	.63(.52)
DM frac in groups‡	.15(.23)	.14(.21)	.23(.38)	.27(.50)

quantities in () are for no-breakup case
* number of halos in the box
** number of overmerged halos
† fraction of halos which are in groups
‡ fraction of total DM which is inside the mean harmonic radius r_h of groups

For our fiducial boxes, the mean spacing between groups is $\sim 9 - 12$ Mpc for all simulations. However, groups are strongly concentrated along filaments surrounded by large voids (see BHNPK), so that the distance to the nearest neighboring group is generally smaller: 5 Mpc on average, and 4.7 Mpc median.

Figure 3 shows, for full boxes cut below $\delta\rho/\rho = 80$, the median group velocity dispersion $v_{\rm gr}$ vs. the fraction of galaxies in groups $f_{\rm gr}$ as D_0 is varied. It is the real space analog of our favored statistic. All models show a tendency for higher $v_{\rm gr}$ at low $f_{\rm gr}$, when only increasingly dense cores are selected. CHDM groups are much "cooler" than CDM groups, and CDM1 groups are significantly "hotter" than CDM1.5 groups.

Figure 4 shows the average distribution of mass around box groups in all simulations. Figure 4(a) also shows separate densities of the hot and cold fractions for CHDM$_2$. Since virial radii r_h ranged from ~ 0.6 to ~ 8 Mpc, we minimized smearing in distance by binning the mass about each group as a function of the non-dimensional radius r/r_h. The result is an unweighted average for all groups. The profile is approximately exponential $\rho \propto e^{-r/r_h}$. Beyond the core, a power law of slope -2 (i.e. an isothermal sphere) fits the CHDM curves reasonably well. Simulations of galaxy clusters in a range of cosmologies (Crone, Evrard and Richstone 1994) find $\Omega = 1$ cosmologies also show an isothermal DM distribution, while $\Omega \simeq 0.2$ cosmologies do not. CHDM$_2$ densities are $\sim 10-15\%$ higher than CHDM$_1$. CDM densities, shown in Figure 4(b), are also exponential. Groups made of halos over $(\delta\rho/\rho)_{cut} = 156$ have densities on average $\sim 70\%$ higher than those made of halos over $(\delta\rho/\rho)_{cut} = 80$. The density distribution shows no detectable change of slope beyond the radius defined by the halos. Indeed, out at $2r_h$ the density is still an order of magnitude above the critical density. This mass is likely still strongly bound to the group. Figure 5 shows the cumulative mass for the fiducial boxes. Note that there is still a significant amount of mass from $r_h < r < 2r_h$ which appears to be part of the groups; $M(2r_h)$ is $2.7M(r_h)$ for CHDM and $2.0M(r_h)$ for CDM. CDM box groups appear to contain a larger fraction of their DM mass at small r. Figure 5(b) shows the cumulative mass density distribution around sky groups. Note that smearing due to grouping in redshift space all but erases the differences in the $M(r)$ trends between simulations. Sky groups are on average much brighter and more massive than box groups, due to the magnitude limit. These more massive groups have an even broader distribution of surrounding dark matter, and $M(2r_h)/M(r_h)$ is $2.8-3.0$ for all simulation groups.

In the Cold + Hot Dark Matter picture, the cold particles fall into gravitational potential wells first, later to be followed by the hot particles after they cool. This offset in time continues until the present and leads to a higher fraction of cold particles in the cores of the groups, as shown in Figure 6. Interestingly, the CHDM$_1$ simulation's higher power on large scales seems to lead to a higher contrast between cold and hot densities, perhaps through earlier collapse and higher concentration of cold particles today. Data inside 195 kpc is unresolved and not plotted.

5. Merging the CfA1 Catalog

Several issues need to be considered before we can make realistic comparisons between simulations and CfA1 data. First is their differing spatial resolution. The galaxy identification scheme requires that a cell identified as a galaxy be at a local density maximum. Thus, the nearest possible neighboring halo will be two cells away. We require the CfA1 data to show the same resolution, on average, before we can reliably identify equivalent groups. We therefore attempt to merge CfA1 galaxies which are sufficiently close. In NKP94 we used a simple merging scheme which assumed an isotropic orientation for the separation vector between all galaxies within a group and merged all galaxy pairs separated on the sky by less than $2r_{cl}/\pi$, where r_{cl} is the spacing between nearest possible neighbors averaged over the positions of all possible nearest neighbors, and $2/\pi$ projects this onto the sky. The isotropic assumption is actually incorrect. Isotropy would only be true if the depth of the group were as small as r_{cl}. Here we perform a more careful merging which more closely models the resolution of the simulations. As such merging may be important for future comparisons between simulations and observations, we describe it in some detail below.

Consider a simulation cell tagged to be a galaxy halo. The nearest possible neighboring halo will lie on a cube 5 cells on a side ("neighbor cube"), such that inside this cube the 26 cells immediately adjacent to the central halo in question cannot be halos. To get an estimate of the mean separation between nearest neighboring halos, one could simply average the lengths of the separation vectors between the parent halo and all cells on the neighbor cube. To improve this estimate, we first weight each cell on the neighbor cube by the frequency that it is actually occupied in the simulations. For example, we find the closest neighbor cells, i.e. those offset from the central cell in one coordinate only, are $\sim 36\%$ more likely to be occupied than those on the edges and corners (average over all simulations). The resulting mean 3D separation between closest neighbors is $r_{cl} = 0.509$ Mpc . Now consider two concentrations of dark matter within the simulations. If the separation r between the centroids of these concentrations is $r < r_{cl}/2$, our halo finder will define these as a single, merged halo. If $r > r_{cl}/2$, they will be identified as two halos separated by r_{cl}. Thus, one should seek to merge CfA1 galaxies closer than $r_{cl}/2$ (those between $r_{cl}/2$ and r_{cl} are balanced, on average, by those between r_{cl} and $1.5r_{cl}$).

The last complication is that for real galaxies only sky-projected separations are known. Clearly, group members in the simulation may be arbitrarily close together on the sky. The probability that they are actually close in 3D depends on the depth of the group relative to the pair's separation on the sky. Since candidate mergers will have sky separations of order $r_{proj} \leq 0.25$ Mpc and a typical group's depth is of order $1.5 - 2$ Mpc , the probability of merging is actually low and the isotropic estimate of NKP94 results in too many CfA1 mergings. To model the probabilities properly, we constructed a suite of artificial groups of various sizes (parameterized by the mean pairwise separation of the member galaxies r_p), with randomly positioned galaxies. Their density followed roughly the density of halos in groups; an $r < 0.55$ Mpc isodensity core surrounded by a $\rho \propto r^{-2}$ envelope, truncated so the resulting group boundary had an aspect ratio of 1:1:2 with the long axis along the line of sight. This elongation corresponds to the observed median aspect ratio of simulation groups in redshift space at the fiducial link parameters. We used these simulated groups to calculate the probability $P_{merge}(r_p, r_{proj})$ that the pair should be merged by tallying those pairs close in 3D as well as in sky projection. The resulting probability curves vs. r_{proj} could be well fit with a set of second order polynomials. The output of this exercise was then an interpolation table of polynomial coefficients for each of 10 r_p's spanning the range of observed CfA1 group r_p's. For a median sized group, two galaxies separated on the sky by $r < r_{cl}$ had a probability for merging of order 0.1. We then looked at each N93 CfA1 group and binary galaxy pair, rolled the dice, and either merged or left alone each pair with $r_{proj} < r_{cl}$. After several trials, we adopted one which was typical, resulting in 39 group members and 5 binaries being merged. Note that this is only 2% of the galaxies.

6. Constructing Sky Projected Redshift Catalogs

An unavoidable problem in comparing all numerical simulations of large scale structure with real observations is how to assign luminosities to masses. How nature does this is still understood only in outline, and it is not obvious how our 1-cell masses should be assigned optical luminosities. We have therefore made the simple but plausible assumption that the baryonic mass in each 195 kpc ^{3} cell turns into stars in such a way that the resulting luminosity function $\phi(L)$ has a Schechter form similar to that of the merged CfA1 catalog, and that galaxy luminosity increases monotonically with galaxy mass.

In solving for the luminosity function of the CfA1 catalog, Nolthenius (1993) assigned distances to each galaxy based on the Burstein-Faber (1987) flow model. Others have used a Virgocentric infall model. For our comparisons here, it is only important that $\phi(L)$ be defined in an equivalent way between the simulations and observations. We therefore assumed simple, unperturbed Hubble flow distances $D = V/H_0$ (except that radial velocities $V < 300$ km s^{-1} are set to $V = 300$ km s^{-1}). The luminosity function was assumed to be of

Schechter (1976) form, and the parameters α and M^* for the merged CfA1 catalog were determined using a code based on the inhomogeneity-independent method of deLapparent, Geller & Huchra (1989, DGH). ϕ^* was constrained by requiring that the appropriate integral over the solid angle A inside $V = 12000$ km s^{-1}

$$N = \frac{A\phi^*}{{H_0}^3} \int_{300}^{12000} v^2 \Gamma(\alpha + 1, \lambda(V))\, dv \tag{1}$$

$$\lambda(V) = L_{lim}(V)/L^* = \text{dex } \left[.4(M^* - 14.5 + 25 + 5\log(\frac{V}{H_0}))\right]. \tag{2}$$

produce the observed number of galaxies N in the CfA1 catalog. $L_{lim}(V)$ is the luminosity of a $m = 14.5$ galaxy at redshift V, and V/H_0 is in Mpc. Our ϕ^* constraint differs from the maximum liklihood methods (e.g. Efstathiou, *et al.* 1988) or that of DGH, yet reproduces their unmerged CfA1 ϕ^*'s well.

Generating the sky catalogs was an interative procedure. First we made fiducial groups from the full box using 3D information. The $\sim 8-10$ most massive clusters ("Virgo"'s) in the CHDM$_1$ box were centered about, and within a factor of $2-3$ of, the adopted Virgo mass of $M_{virgo} = 2.5 \times 10^{14}$ $M_\odot$. Corresponding CHDM$_2$ clusters were less massive. We placed an observer on a "home galaxy" 20 Mpc away from one of these clusters. We then chose luminosity assignment parameters $(\delta\rho/\rho)_{cut}$, the lowest halo mass retained, and Schechter parameters α and M^*, and generated N Schechter distributed magnitudes to be paired with the N halos above $(\delta\rho/\rho)_{cut}$ in such a way that the luminosity as a function of halo mass L(M) rose monotonically. We then observed the box, keeping galaxies above apparent magnitude m=14.5 out to $V = 12000$ km s^{-1}. Radial velocities v_r in direction $\hat{r}$ at distance r were calculated from

$$v_r = (\vec{v} - \vec{v_0}) \cdot \hat{r} + H_0 r, \tag{3}$$

where the the observer's home galaxy has peculiar velocity $\vec{v_0}$. Because of redshift distortion, the observed α is ~ 0.4 steeper and the observed M^* is ~ 0.4 brighter than the input values for our simulations, requiring us to iterate until we fit the observed CfA1 α, M^*, and projected galaxy density. These L(M) parameters were then used to do a random search in the CHDM$_1$ and CHDM$_2$ boxes for candidate home galaxies which satisfied the following conditions: (a) the local galaxy density in redshift space ($V < 750$ km s^{-1}) was within a factor of 1.5 of the merged CfA1 galaxy density (though still usually on the low side), and (b) the closest Virgo-sized cluster was 20 Mpc away in distance. Among these candidates we chose 6 which were distributed as widely through the box as possible. Since the CDM1 and CDM1.5 boxes had the same initial random wave amplitudes as CHDM$_1$, the locations of large clusters and filaments were essentially the same as for CHDM$_1$. In order to further remove noise due to cosmic variance when comparing the different cosmological models, we therefore chose the 6 viewing locations within CDM1 and CDM1.5 to be on the halos nearest to the viewing halo coordinates found for CHDM$_1$. The resulting home galaxies had an average peculiar velocity of $V_{pec} \simeq 800$ km s^{-1}. A final iteration then adjusted our L(M) parameters slightly so that the Schechter functions of the 6 sky catalogs matched that of the CfA1.

Since we retained all data out to $V = 12000$ km s^{-1} and our 100 Mpc box corresponds to only 5000 km s^{-1} for $H_0 = 50$ km s^{-1}Mpc^{-1}, this required periodically replicating each simulation box on all of its faces. However, at greater distances only the brightest galaxies are retained in our magnitude limited catalogs, so the replica boxes are sampled sparsely. The effect of replication should be negligible, since we are concerned with the median properties of groups. It simply means we are taking medians from a smaller sample of unique groups than would otherwise be the case for a volume of this size.

We found that setting the apparent magnitude m to 14.5 for a halo of $(\delta\rho/\rho)_{cut} = 80$ (CHDM) and 156 (CDM) at $V = 300$ km s^{-1} (thus retaining all CfA1 data) produced a total density of halos $\sim 0 - 20\%$ above that of the merged CfA1 catalog. The difference between $(\delta\rho/\rho)_{cut} = 80$ and 156 has only a minor effect on the sky catalogs since the imposition of an apparent magnitude limit means that only a few percent of sky catalog galaxies have $\delta\rho/\rho < 156$. (By contrast the volume limited box sample is dominated by low mass halos and groups, due to the steep mass function.) Random culling down to CfA1 density then yielded our final sky catalogs. The 10.384 sr catalogs each had 9204 galaxies inside a redshift of $12,000$ km s^{-1}. All 2.66 sr catalogs had 2358 galaxies within $12,000$ km s^{-1}.

To approximately simulate boundary effects yet still retain a large sample of the sky, we excised a 20° wide "zone of avoidance", leaving a solid angle of $A = 10.384$ sr. This boundary does not truly mimic that for the CfA1, since the CfA1 boundary length per unit area is much higher. To test the sensitivity of our results to this effect, we generated a second set of sky catalogs which had the local "virgo" rotated to its CfA1 right ascension and declination, and the CfA1 latitude and declination limits imposed, giving a solid angle of $A = 2.66$ sr. Our results turn out to be quite insensitive to these boundary and sample size differences.

While we believe enforcing these viewpoint selection criteria is important for making proper comparisons, these critia do have a small but systematic effect the final group properties. To show this, we substituted 6 randomly chosen galaxies as viewing locations for CHDM1, with the only requirement that they be at least 1/3 the size of the Milky Way, as best we can estimate. One of these random points was in a dense region only 9, 20 and 20 Mpc away from the nearest 3 Virgo-sized clusters. The other 5 were in less dense areas with the nearest Virgo-sized cluster $32 - 50$ Mpc away. After tuning the L(M) parameters as described above, the random viewpoint results gave systematically higher $f_{\rm gr}$ by $\sim 1 - 3\%$. $v_{\rm gr}$ did not systematically change except at the two highest $f_{\rm gr}$, where it was ~ 30 km s^{-1} lower.

The periodic boundary conditions mean that most bright galaxies appear more than once in the final sky catalog. On average, each galaxy appears $\sim 3 - 4$ times in the full 10.384 sr catalogs, and ~ 1.8 times in the 2.66 sr catalogs. This would be a concern for any study focusing on large scale structure, since in this case the sample size would not even approach being "fair". However, we focus on median group properties and these should be insensitive to this level of replication.

Our adopted breakup procedure produced a high fraction of fragments in the sky catalogs: 27% (CHDM), 50% (CDM1.5) and 61% (CDM). Using breakup Method 1, a high percentage of sky catalog galaxies were also fragments of overmergers: 27% for CHDM and 70% for CDM. These percentages were only 11% (CHDM), 37% (CDM1.5) and 46% (CDM) for Method 2, which had many more low mass fragments.

The "true" mass of a DM halo is not a well defined concept. The DM around a simulation galaxy merges smoothly with the DM in a group, which merges smoothly with a general background. We believe it's plausible, however, that regardless of how one chooses to (consistently) define it, the DM mass will have a monotonic relation to our 1-cell mass. If we further assume that DM halo mass and blue luminosity are monotonically related, then our luminosity assignments should be insensitive to the detailed relations. Nevertheless, the relation between DM mass and 1-cell mass is likely to be non-linear, so that one would be suprised if a constant M/L resulted from our luminosity prescription, even if in some sense M/L were a constant for real galaxies. Figure 7(a) shows M/L for all of our sky catalogs. While it diverges upward at the smallest masses (which comprise only a tiny fraction of the sky catalog members), for the great majority of galaxies $M/L \sim 10$ for CHDM and $M/L \sim 20 - 25$ for CDM. (Other luminosity assignment methods have been tried in earlier dissipationless simulations. For example, Gelb (1992) uses the Tully-Fisher and Faber-Jackson relations. He too finds a luminosity function which is roughly constant over intermediate masses and deviates at the low and high ends.) When the massive halos are broken up, M/L for the fragments ends up lower than for the no breakup case. Figures 7(b) and 7(c) show the distribution of 1-cell masses in the sky catalogs. Note that breaking up the overmergers changes the mass distribution substantially. A strong peak occurs for the fragments, just below the cutoff mass $M_{bu} = 7 \times 10^{11}\ M_\odot$. This is especially true for CDM, which has higher mass halos than CHDM.

Table 3 shows the characteristics of the resulting sky catalogs. Note that merging lowers CfA1's luminosity density and brightens M^* compared to the unmerged CfA1, though the effect is very slight with only 44 mergers. This is because new mergers which would ordinarily be formed from galaxies with $m > 14.5$ are missing from the CfA1 sample.

Table 3. Sky Catalog Parameters

Catalog	$V_{pec}^\dagger$	α	M^*
CfA1 (full)	-	-1.27	-21.08
CfA1 (merged)	-	-1.26	-21.10
CHDM1	852±99	∼-1.24	∼-21.23
CHDM2	595±247	∼-1.20	∼-21.23
CDM1.5	700±127	∼-1.19	∼-21.19
CDM1	991±169	∼-1.19	∼-21.19

† peculiar velocity of home galaxy and 1σ range over 6 viewpoints

To test the sensitivity of our comparisons to luminosity assignment, we considered two alternate methods. Both of these alternates were performed on the no breakup cases only. The first was suggested by J. Ostriker (private communication 1993). Cen and Ostriker's (1993; CO) hydrodynamic simulations lead to a relation between CDM total density ρ_{tot} (dark matter + gas/stars) and collapsed baryonic density ρ_{baryon} (presumably stars). If $\hat\rho \equiv \rho/\bar\rho$ then CO find $\log(\hat\rho_{\rm baryon}) = A + B\log(\hat\rho_{\rm tot}) + C\log^2(\hat\rho_{\rm tot})$. B and C depend mildly on smoothing scale R. In our simulations, we considered $\hat\rho_{tot}$ the density within a 3-cell cube centered on the halo, giving a smoothing scale of R=0.6 Mpc. Extrapolating CO's trends for coefficients B and C down to $R = 0.6$ Mpc then gave $B = 2.3, C = -0.20$. We assume M_{baryon}/L is a constant. The normalization A was set to insure that after magnitude limiting $(\delta\rho/\rho)_{cut} = 80$ galaxy halos to $m = 14.5$ for all simulations, the number of galaxies retained was similar to the merged CfA1 catalog, giving A=6.23 for both CHDM models and A=5.8 for both CDM models. This relation, with our distribution of masses, produces a Schechter-like luminosity function, but with M^* two and a half magnitudes too bright (~ -23.5), and α too steep (~ -1.7). The resulting halo M/L (see Figure 7(a)), attaches much higher L's to massive galaxies, relative to the Schechter prescription. This leads to a disproportionate number of distant, luminous galaxies at the expense of those closer.

The second method uses the blue Tully-Fisher relation from Fouque, *et al.* (1990). We assume the cell mass M is gravitationally bound so that a circular velocity may be defined

$$V_{circ} = \sqrt{\frac{GM}{r}}. \tag{4}$$

The Fouque, *et al.* Tully-Fisher relation then defines the absolute B magnitude M_B of the galaxy as

$$M_B = -5.5\log(V_{circ}) - \beta \tag{5}$$

where β is a calibration. This is equivalent to $M^{1.1}/L$ = const. Using the Fouque, *et al.* value of $\beta = 8.0$ corrected by -0.37 for average internal extinction in spirals and by -0.29 to put on the Zwicky system results in over an order of magnitude too few galaxies surviving the $m = 14.5$ limit! This isn't too suprising. Our halos are 200 kpc across, ~ 10 times bigger than the visible size of a typical spiral, and the relation between our 1-cell mass M and the observational $M(r \sim 10$ kpc $)$ is not known. Also, we no doubt systematically underestimates mass at this size scale due to our limited force resolution. We therefore adopt a β which best matches the merged CfA1 galaxy density of 886 sr^{-1}. The resulting halo M/L attaches much higher L's to low mass galaxies, relative to the Schechter prescription. This leads to catalogs with a disproportionate number of nearby galaxies and strongly favors fainter, lower $v_{\rm gr}$ groups.

7. Tuning the Grouping Algorithm Link Parameters

Conventionally, a group is thought of as a gravitationally bound collection of perhaps 3 to 50 galaxies which has collapsed. For our purposes, we consider a group more generally as any set of galaxies which satisfy a linking criteria on the sky and in redshift. Any valid cosmological model must be able to produce a set of such groups whose typical properties are in good agreement with identically selected real groups *for any link lengths.* It is not necessary that the groups be bound, let alone collapsed or virialized. Nevertheless, it is also of interest to identify that set of groups which is closest to satisfying the conventional definition. While minimizing contamination by interlopers is important, for e.g. estimating Ω from the M/L method (see §8), it is even more important that the collective group properties most closely match those for groups of comparable overdensity identified using full 3D information. The corresponding link criteria which make these most realistic groups are referred to below as the "fiducial" links.

We use the adaptive grouping algorithm of Nolthenius (1993), which is a modification of the original friends-of-friends algorithm in redshift space given by Huchra and Geller (1982). Galaxies at redshift V km s^{-1} are linked if the separation on the sky is less than D_L and their difference in redshift is less than V_L, where

$$R^{sky}_{M.I.S.}(V) = \frac{2}{\pi}\phi^*\Gamma\big(1+\alpha,\lambda(V)\big), \tag{6}$$

$$D_L(V) = D_n R^{sky}_{M.I.S.}(V_0)\left(\frac{\Phi(V_0)}{\Phi(V)}\right)^{1/2}\left(\frac{V_0}{V}\right)^{1/3}, \tag{7}$$

$$V_L(V) = V_5\left(\frac{\Phi(5000\ \mathrm{km\ s^{-1}})}{\Phi(V)}\right)^{1/3}. \tag{8}$$

$\lambda(V)$ is given by equation (3). V_0 is an arbitrary redshift distance used for scaling, here taken to be 1000 km s^{-1}. D_n, an input parameter, is the sky link expressed as a fraction of the sky-projected mean interparticle spacing $R^{sky}_{M.I.S.}$. $\Phi(V)$ is the integrated galaxy luminosity function above the apparent magnitude limit visible at redshift V. D_L is designed to insure that a group at the minimum number density contrast just meets the selection criteria at all redshifts (see NW). We scale V_L with the mean interparticle spacing. We chose this scaling, rather than the linear NW scaling (whose resulting groups $\frac{M}{L}(V)$ trend turned out to match slightly better to groups selected using full 3D information at the same overdensity limit), simply because it produced a flatter distribution of group M/L with distance. We have also run cases with the NW version of the algorithm and find negligible differences in group properties and no change in our conclusions. For the purposes of comparing models, either $V_L(V)$ can be used. We parameterize the size of the link in redshift with $V_5 \equiv V_L(5000$ km s$^{-1})$.

Consider a collection of galaxies which are close together in redshift space. Their rms velocities about their mean peculiar velocity will govern how elongated along the line of sight this group appears, providing a diagnostic for determining the optimal redshift link V_5. Define the aspect ratio A_z of this "finger of God" seen in redshift space as $A_z = V(\Delta RA + \Delta Dec)/2\Delta V$, where Δ refers to the maximum extent of the group in right ascension, declination, and line of sight velocity. Define a group's aspect ratio A_r in real space as $A_r = V(\Delta RA + \Delta Dec)/2\Delta(V - V_{pec})$. Note that A_r can be defined only for simulation groups, since the galaxies' true distances must be known. Because groups are often poorly isolated from their neighbors and because of random peculiar velocities, the true depth of groups picked out in redshift space will inevitably be larger than the sky-projected group size; groups will be elongated along the line of sight. This is not the familiar "finger of God" one sees in redshift maps, it is its counterpart seen in real space. This elongation would, on average, disappear if one could assign memberships with perfect knowledge of galaxies' true distances. Suppose a set of galaxies close in redshift space has a small rms velocity $v_{\rm gr}$ about their mean peculiar velocity. At V_5 below some threshold, related to the local velocity dispersion, one picks out only groups whose members are within $\sim D_L$ of each other in real space, and A_r is near a minimum (in fact, since there are two sky dimensions and only one depth dimension, there is a tendency for A_r to rise slightly as V_5 approaches zero, when only a fraction of the valid members are being selected). At V_5 below the threshold too many valid group members are excluded, and we refer to this as the "clipped regime". Above the threshold V_5, A_r begins to rise as outliers begin to significantly contaminate the group. High V_5 is then said to produce groups in the "interloper regime". Figure 8 shows A_r vs. V_5 for our simulations (on this and later plots $D_n = 0.36$ unless otherwise noted). The transition between regimes is rather gradual, as groups are poorly separated from neighboring galaxies in all models. The clipped/interloper transition, by this measure, is near $V_5 \sim 200$ km s^{-1} for CHDM and CDM1.5, and about 400 km s^{-1} for CDM1. CDM A_r curves are significantly higher and more steeply sloped than those for CHDM as grouping percolates through the high velocity interlopers from the more diffuse surroundings Note that the minimum A_r does not approach 1, due to the inherent loss of depth information in redshift space. It is possible to force rounder groups and thus smaller A_r's by raising D_n and restricting V_5, but at the cost of rejecting an unacceptably high number of valid members. For comparison, Figure 8 also shows the curves for groups from a 10.38 sr sky catalog made from a 100 Mpc box of Poisson distributed particles with Gaussian random velocities of $\sigma = 200$ km s^{-1} and $\sigma = 350$ km s^{-1}. In these catalogs, there is no coherent clustering to confine the depth dimension, which therefore rises steeply with V_5 even at small scales. Note by comparison that even the CDM catalogs show significant coherent motion by their more slowly rising curves.

We believe the most relevant measure here is to ask what V_5 is required in order to produce the same median $v_{\rm gr}$ as is seen in groups selected using full 3D information on our fully corrected simulations at the same fiducial $D_n = 0.36$. This is shown in Table 4. This is $V_5 \simeq 350 - 400$ km s^{-1} for CHDM, $V_5 \simeq 550$ km s^{-1} for CDM1.5, and 670 km s^{-1} for CDM1. If no breakup is performed, $v_{\rm gr}$'s are very similar, but generally require somewhat lower V_5's to match up with 3D groups.

Table 4. V_5 Link Best Matching 3D Groups' $v_{\rm gr}$'s*

simulation	$\rm CHDM_2$	$\rm CHDM_1$	CDM1.5	CDM1
V_5	368(375)	397(348)	571(485)	669(576)
$\langle v_{\rm gr} \rangle_{med}$	121(119)	123(119)	183(162)	225(202)

$*V_5$ link giving redshift space groups with the same $\langle v_{\rm gr} \rangle_{med}$ as 3D groups, at fiducial $D_n = 0.36$. All in km s^{-1}, no breakup case is in parentheses

Finally, we note that the fraction of galaxies grouped $f_{\rm gr}$ will at first rise steeply with V_5 as new members are rapidly added from the dense region containing the group. The $f_{\rm gr}$ (V_5) curve will then show a distinct drop in slope as it enters the interloper regime, when essentially all valid members have been added and primarily outliers and interlopers from the lower density surroundings are incorporated at higher V_5. This transition occurs near $V_5 \sim 350$ km s^{-1} for both CHDM simulations, and near $V_5 \sim 600$ km s^{-1} for both CDM simulations (see §9). As these values are also representative of the other measures above, we adopt $V_5 = 350$ km s^{-1} as the fiducial redshift link for CHDM, and $V_5 = 600$ km s^{-1} as that for CDM.

8. The Dynamical State of the Groups and Implied Ω

To assess the dynamical state of groups, we calculated the number- weighted virial mass estimator for 3D-selected box groups. This is the M_{vir} appropriate if the mass is dominated by a dark matter background distributed like the galaxies, and is given by

$$M_{vir} = \frac{2\sigma r_h}{G}, \tag{9}$$

where, for a group of n galaxies, $\sigma = (n-1)^{-1}\sum_i(\vec{v_i} - <\vec{v}>)^2$ is the 3D rms velocity about the mean velocity $<\vec{v}>$ of the group. We then compared this to the true mass M_{DM} in dark particles within the mean harmonic radius r_h, where

$$r_h = \frac{n(n-1)}{2\sum_{i<j}^{n} r_{ij}^{-1}}, \tag{10}$$

and r_{ij} is the separation between galaxies i and j.

Figures 9(a) and 9(b) show scatter plots of group log M_{vir} vs. log M_{DM} for the no breakup and breakup CDM boxes, respectively. Many of the no breakup CDM groups appear to be unbound, with the median M_{vir}/M_{DM} biased as high as 2.0 for CDM1. Breaking up the overmergers reduces this bias significantly. This effect is not due to the higher minimum halo overdensity ($(\delta\rho/\rho)_{cut} = 156$ vs. $(\delta\rho/\rho)_{cut} = 80$ for CHDM), since it is also clearly seen in the $(\delta\rho/\rho)_{cut} = 80$ CDM boxes. It also appears equally strong when only better sampled groups (at least 10 members) are considered. The explanation is that many small groups do not appear in the no breakup cases, because of overmerging. Recall that a halo is defined as a local density maximum, requiring neighboring cells to be relatively underdense. Also, a group must contain at least 3 members to be identified. The overmerged CDM halos often dominate their surroundings to such an extent that too few other local density maxima can be defined, preventing a group from being identified. When these are broken up, new groups result. These groups turn out to have M_{vir} close to M_{DM}, on average, so that their inclusion lowers the median M_{vir}/M_{DM} nearer to unity. This issue affects CDM much more than CHDM, since CDM has ~ 4 times as many overmergers (see Table 2).

Note that the breakup case in Figure 9(b) still has these high M_{vir}/M_{DM} groups. Correlating high M_{vir}/M_{DM} in the no breakup CDM box with other group properties, we found that these groups tend to be poorly sampled (all groups with $M_{vir}/M_{DM} > 30$ have 5 or fewer members), have higher than average size, low density contrasts, high peculiar velocities, and tend to lie in denser regions. Using computer visualization and color-coding groups by their M_{vir}/M_{DM} shows these groups tend to be small and lie near the outer parts of dense clumps and filaments. Putting this all together suggests these groups may have experienced tidal shearing sufficient to unbind them. Other recent work supports this idea (e.g. Mamon 1994). Some high M_{vir}/M_{DM}'s are no doubt simply due to the inherent high noise in M_{vir}. Figure 9(c) shows M_{vir} vs. M_{DM} for groups from all 6 fiducial linked 10.38 sr catalogs. Note that the high V_5 required to match median 3D $v_{\rm gr}$'s nevertheless produces a tail of spurious groups with high $v_{\rm gr}$, little mass within r_h, and hence high M_{vir}/M_{DM}. The high V_5 link essentially produces a floor for M_{vir}, which is unreasonably high for the smallest groups. This is a general feature of all group catalogs linked in redshift space (see Geller & Huchra 1983; HG, RGH). Banding is due to using only a 10% sample of the dark particles; the smallest (spurious) groups contain only a few dark particles.

The corresponding plots for the CHDM boxes are in Figure 10. CHDM box groups appear little changed by breakup, partly because they contain only a quarter as many overmergers, and partly because they show lower random velocities on small scales. Note in Figure 10(c) that the lower V_5 level appropriate for CHDM significantly reduces the problem of spurious high M_{vir}/M_{DM} groups. If, as the present study suggests, a low V_5 is also appropriate to the real universe, the problem of contamination and spurious groups may not be as severe as seen in e.g. Geller and Huchra (1983)

Note that massive groups ($M > 10^{14}\ M_\odot$) in all boxes appear to define a narrow M_{vir}/M_{DM} which appears clearly virialized, yet which lies systematically higher than the virialization line. There are two biases towards high M_{vir}/M_{DM} which likely account for this, and which are present for all groups. First, the r_h sphere within which particles are counted is centered on the mean position of the halos. Centering instead on the dark particle concentrations lowers M_{vir}/M_{DM}. We confirmed this by centering on the mass-weighted mean position of the particle condensations within $1.5 r_h$ of the groups and found it raises M_{DM} such that the median M_{vir}/M_{DM} is lowered by 7%. Second, many groups, especially the larger ones, are elongated density enhancements along filaments. Counting within a sphere will include spurious low density areas along the "equator" while missing the high density regions along the "poles". If this bias is $\sim 30\%$ in M_{DM}, which seems plausible (see group geometries in the video sequence of BHNPK), then $M > 10^{14}\ M_\odot$ groups in the breakup boxes appear to obey the virial theorem, on average, in all simulations.

We also determined several measures of the "velocity bias" b_v (Carlberg & Couchman 1989): the velocity of the galaxies compared to that of the underlying cold dark matter particles. The global velocity bias b_v^{global} is defined by a sum over the i galaxies and j DM particles within the box $b_v^{global} = \left(\sum_i v_{i\ gal}^2 / \sum_j v_{j\ DM}^2\right)^{1/2}$. The results are summarized in Table 5 below and shown in Figure 11, using all halos with 1cell mass above $1.5 \times 10^{10}\ M_\odot$. Figure 11 shows that b_v is fairly noisey for low and moderate mass groups (and more so for CDM), but is confined quite closely about the global median for the virialized $Log M_{DM} > 13.8$ clusters. The velocity bias within a group's r_h is defined over the i galaxies and j DM particles as $b_v^{grp} = \left(\sum_i (\vec{v}_i - <\vec{v}_{gal}>)^2 / \sum_j (\vec{v}_j - <\vec{v}_{DM}>)^2\right)^{1/2}$, where $<\vec{v}_{gal}>$ is the mean unweighted galaxy velocity and $<\vec{v}_{DM}>$ is the mean DM particle velocity within the group. CHDM galaxies show a moderate bias but only within groups. CDM1.5 is moderately biased both globally and inside groups. CDM1 galaxies, however, show essentially no bias, in or out of groups, in agreement with $b_v^{global} = 0.94$ from Katz, Hernquist, & Weinberg's (1992) hydrodynamic simulation. KNP describes other velocity measures in the full boxes.

Table 5. Velocity Bias

simulation	b_v^{global}	$< b_v^{grp} >_{avg}$	$< b_v^{grp} >_{med}$
$CHDM_1$	0.94(0.90)	0.85(0.78)	0.78(0.76)
$CHDM_2$	0.93(0.89)	0.76(0.69)	0.73(0.71)
CDM1.5	0.83(0.79)	0.82(0.74)	0.78(0.79)
CDM1	0.96(0.90)	1.08(0.92)	1.00(1.06)

No breakup case is in parentheses

The standard method for estimating Ω from bound virialized groups is to assume all galaxies have the same mass-to-light ratio M/L, given by the median M/L for groups, then integrating over the luminosity function to get the mass density (Kirschner, Oemler & Schechter 1979). For a Schechter luminosity function this means

$$\Omega = \frac{8\pi G}{3H_0^2}\langle\frac{M}{L}\rangle\phi^* L^* \Gamma(\alpha+2). \tag{11}$$

Table 6 shows the resulting inferred Ω using the fiducial grouping algorithm parameters and Schechter L(M) for the sky catalogs. Means and standard deviations are over the six viewpoints. Using the Cen-Ostriker L(M) gives very similar Ω's. No $N(z)$ corrections (see §9) were applied to get $< M/L >_{med}$ since group M/L is virtually independent of redshift.

Table 6. Ω from Group M/L Method

catalog	V_5	$avg < M/L >_{med}$	Ω
CfA1 (full)*	350	62	0.06
CfA1 (merged)	350	79	0.08
$CHDM_1$	350	$128 \pm 18 (150 \pm 11)$	$0.13 \pm .02 (0.14 \pm .01)$
$CHDM_2$	350	$134 \pm 18 (126 \pm 13)$	$0.12 \pm .020 (0.11 \pm .01)$
CDM1.5	350	$158 \pm 21 (178 \pm 12)$	$0.17 \pm .02 (0.18 \pm .02)$
CDM1	350	$171 \pm 27 (183 \pm 44)$	$0.18 \pm .02 (0.19 \pm .04)$
CfA1 (full)*	600	119	0.12
CfA1 (merged)	600	152	0.15
$CHDM_1$	600	$188 \pm 25 (207 \pm 21)$	$0.19 \pm .02 (0.19 \pm .01)$
$CHDM_2$	600	$212 \pm 30 (193 \pm 21)$	$0.20 \pm .03 (0.17 \pm .02)$
CDM1.5	600	$322 \pm 35 (347 \pm 32)$	$0.34 \pm .03 (0.34 \pm .04)$
CDM1	600	$343 \pm 63 (395 \pm 34)$	$0.36 \pm .06 (0.40 \pm .03)$

* full, unmerged CfA1 catalog.
No-breakup results in parentheses

Note that all of our $\Omega = 1$ simulations yield low observed Ω's. Thus, these simulations show a feature which has been seen in real data for over a decade and which has been used by some to argue for a low Ω universe: How can Ω be 1 if bound groups and clusters consistently account for only $\simeq 10-20\%$ of the mass? Three factors can explain this discrepancy. (1) Only the mass within the mean harmonic radius r_h of the groups is measured by the virial estimator. We saw in Figure 4 that density falls approximately like an isothermal sphere outside group cores, and that this continues to at least $2r_h$ (about midway to the next nearest group, on average), where the cumulative mass $M(2r_h) = 3M(r_h)$ (see Figure 5(b)). This additional factor of 3 in mass is almost certainly bound to the groups, since $\rho(2r_h) \simeq 4\rho_c$(CHDM) to $10\rho_c$(CDM), and infall is still occurring. The fraction of the total mass in the box which lies within the r_h sphere of fiducial linked groups is only $\sim 15-25\%$, as shown in Table 7. Most mass lies outside of groups.

Table 7. Mass, Volume Fraction Within Box Groups

simulation	N^*_{grps}	f^{**}_{mass}	$f^{\dagger}_{volume}$
$CHDM_1$	575	0.14	0.0086
$CHDM_2$	640	0.15	0.0072
CDM1.5	736	0.23	0.0038
CDM1	617	0.27	0.0026

* Number of groups in box
** fraction of box mass which is within r_h of groups
† fraction of box volume which is within r_h of groups

(2) The relevant virial velocities appropriate for measuring the local mass are actually those of the individual particles, not the galaxies. As already seen in Table 5 and Figure 11, the median velocity bias within CHDM groups is ~ 0.75. Since $M_{true}/M_{vir} \propto 1/b_v^2$, this contributes another factor of 1.8 to the Ω estimate. (3) We've also assumed spherical symmetry in Figures 4 and 5. The true shape of groups is more elongated, as structure is still quite stringy at this point in the evolution. The inappropriately counted matter along the "equator" of the groups is of lower density and fails to compensate for the uncounted high density regions missing above the "poles" by perhaps a factor of 1.3. The product of these factors (~ 7) approaches an order of magnitude. While factor (1) above is closely related to the fact that the galaxies are a biased (b=1.5) tracer of the DM, it seems likely that the less clustered hot DM, especially that filling the voids (see BHNPK) will additionally bias M/L to the low side. Together, these factors show that, within the assumptions of this method, an observed $\Omega \simeq 0.1$ can indeed be consistent with a true $\Omega = 1$ universe. In fact, virial estimates appear to be underestimates of the true mass in real clusters as well. At present, gravitational lensing appears to be the most reliable method of measuring total masses, and several studies show that typically the resulting total masses of clusters out to just beyond the optical radii is roughly a factor of 2 or 3 higher than virial estimates (e.g. Fahlman, *et al.* 1994). FEWS also find that their high resolution hydrodynamical simulations of a rich cluster give a similarly severe underestimate of the true Ω. There are other, more subtle problems with the M/L method. For example, the stellar populations of groups and especially clusters is well known to be older and have lower M/L than is typical for the field galaxy populations.

Our Ω at $V_5 = 350$ for the CHDM simulations is 50% higher than that for the merged CfA1 at the same V_5: $\Omega_{CHDM} = 0.12$ vs. $\Omega_{CfA1} = 0.08$. The difference is due almost entirely to the difference in the median of the mean harmonic radii of groups (see §10), which enters in the virial mass. Our Ω for the full, unmerged CfA1 is smaller than that of HG's $\Omega \simeq 0.1$ for the nearby, all sky m=13.2 CfA sample, and smaller than RGH's $\Omega \simeq 0.13$. As we've argued here and elsewhere (NW, N93), we prefer a significantly smaller redshift link than that used by these authors, which directly lowers the virial masses. The RGH work is for the CfA Slices, whose group's show a suprising 60% higher $v_{\rm gr}$ than for their equivalently selected CfA1 groups, due to differing sample depth and, to some extent, cosmic variance (see RGH for discussion). Also, their link parameters differ from ours. They used a smaller sky link (giving smaller r_h) but larger redshift link, leading to higher $v_{\rm gr}$'s and net higher M/L's.

When using full 3D information to make sky catalog groups (D_n=0.36 as before), our median Ω's are higher: Ω =0.16(both CHDM), 0.32(CDM1.5), and 0.48(CDM1). Note that the V_5's necessary to match 3D $v_{\rm gr}$'s are, except for CDM1.5, higher than our fiducial V_5, as shown in Table 4. For example, raising CHDM's V_5 from the fiducial 350 to 397 km s^{-1} raises $< v_{\rm gr} >_{med}$ by $\sim 15\%$ and hence $< M/L >_{med}$ and Ω by $\sim 32\%$, lessening the difference between the observationally inferred and true Ω. The same holds for CDM1. Our CDM1.5 estimate of $\Omega \simeq 0.33$ (for Table 4's $V_5 = 572$ km s^{-1}) agrees well with that of Katz, Hernquist & Weinberg's (1992) TREESPH hydrodynamic simulation result of $\Omega \simeq 0.31$ on a much smaller sample.

9. Sky Catalog Results and Comparisons with CfA1

When comparing our sky catalog groups with those of CfA1, there is one more calibration to consider. To maximize the amount of precious observational data used, we retain the full magnitude limited CfA1 catalog rather than a volume limited subset. In a magnitude-limited catalog, both the sky and redshift "friends-of-friends" links scale up with distance, and group properties will change significantly with distance. In particular, more distant groups will be larger, brighter, and have higher $v_{\rm gr}$'s. Median values will therefore be biased if groups are distributed differently with distance between the two datasets. Chance differences in large scale structure will mean, in general, that groups are in fact distributed differently with redshift. Our home galaxy selection criteria do not fully insure that simulation groups are distributed in redshift like CfA1 groups. Equally important is our small box size. Periodic boundary conditions give repeating structures every 100 Mpc = 5000 km s^{-1}, whereas the CfA1 data are actually rather sparse beyond the Virgo Cluster and before the "Great Wall". Figure 12(a) shows the galaxy density vs. redshift for the merged CfA1 and CHDM$_2$ 2.66 sr catalogs. Relative to the CfA1 dataset, CHDM$_2$ remains underpopulated out to $\sim$ 3000 km s^{-1} and overpopulated beyond. The other simulations follow this same pattern. Figure 12(b) shows how the CHDM1, CDM1 and CfA1 2.66 sr groups are distributed in redshift and $v_{\rm gr}$. Relative to the simulations, CfA1 groups are overabundant nearby and scarce beyond $\sim$ 8000 km s^{-1}, since more distant, luminous, and richer groups have higher $v_{\rm gr}$, on average. Not correcting for this "$N(z)$ bias" will lead to overestimating the average or median sizes and $v_{\rm gr}$'s of simulation groups, as well as affect other properties. We've handled this by averaging results for four random subsamples of each simulation grouping run such that, when their redshifts are binned to 1000 km s^{-1} bins, the number of groups vs. redshift $N(z)$ matches that of the CfA1 groups selected at the same links. The median group properties we present below are medians from these subsamples are labelled "$N(z)$ corrected" for clarity.

In Figure 13 we show $v_{\rm gr}$ vs. V_5 at our fiducial $D_n = 0.36$. Here and throughout this paper, the $v_{\rm gr}$'s for CfA1 groups is corrected by subtracting in quadrature the tabulated rms velocity measurement errors for the member galaxies from the raw $v_{\rm gr}$'s. Figure 13 is analagous to Figure 2 in NKP94, which was done at $D_n = 0.47$ to highlight good agreement in CDM1.5 velocities between our work and that of Moore, Frenk and White (1993; MFW). They used a quite different particle-particle-particle-mesh code with better spatial resolution but poorer mass resolution than ours here. This figure emphasizes again one of our main conclusions: CDM groups have much higher internal rms velocities than observed, while CHDM is in good agreement.

It may be suprising to see that the curve for CDM1.5 differs so little from that of CDM1, since numerous studies, including Figure 3 here, show biased CDM has lower velocities. Our global rms peculiar velocities (i.e. for all galaxies within the box) are indeed much lower for CDM1.5 than those for CDM1: 650 km s^{-1} vs. 944 km s^{-1}. Within groups, however, two different effects combine to reduce this difference. First, on small scales ($\simeq$ 1 Mpc), CDM1 rms peculiar velocities are only 30% higher than for CDM1.5. This is true both for the full set of box groups, and for the 3D selected groups from the sky catalogs. Second, while the two CDM simulations' overall number of galaxies differ by $\sim$ 20%, their spatial structures are actually quite similar (BHNKP), and very diffuse. Blurred in redshift space, such structures will group more sensitively with respect to V_5 than will more concentrated and filamentary geometries like that of CHDM. Using the same V_5 = 600 km s^{-1} on both CDM simulations shows that median $v_{\rm gr}$'s differ by only 9% (189 km s^{-1} for CDM1.5 vs. 207 km s^{-1} for CDM1). This same effect is seen in MFW, and our result agrees well with the 8% difference seen between their b=1.6 and b=2.0 CDM median $v_{\rm gr}$'s. Somerville *et al.* (1995) has re-done and corrected the classic CfA1 pairwise velocity dispersion analysis of Davis & Peebles (1983), which includes this effect, and also find relatively poor discrimination between CDM1 and CDM1.5. Finding an observationally accessible statistic which preserves the differences in real space rms velocities within such diffuse structure will be challenging.

It is important to note that the error bars on these and later curves are $1\sigma_{sv}$ ("sky variance") deviations from the 6 different sky catalogs. While our intent was to measure cosmic variance σ_{cv}, in fact these are generally an underestimate. Because of the small size of our box, many of the same groups are seen in most or all viewpoints, (albeit sampled differently due to the magnitude limit). The difference between the CHDM_1 and CHDM_2 curves may be a fairer estimate of cosmic variance, and this is slightly larger than typical σ_{sv}. We have estimated the probability of a 100 Mpc ^{3} box having the power spectrum of CHDM_1 at $\sim 10\%$ (KNP), corresponding to about $1.7\sigma_{cv}$. If so, then the difference between the CHDM_1 and CHDM_2 curves may be a rough estimate of $1.7\sigma_{cv}$ error bars due to cosmic variance.

Another important point is that since CHDM_1, CDM1, and CDM1.5 all have higher power in the longest waves within the box, the abundance and size of large filaments and clusters is higher, and the fraction of galaxies in groups is higher than would be typical. If one believed the CfA1 data were a "fair sample" (but see below), then comparing to CHDM_2 would be more appropriate. To estimate this effect on the CDM curves one can do a rough calibration by looking at the CHDM_2 curves shown here and below and shift the CHDM_1 curve towards it, carrying along rigidly the two CDM curves in the same direction. In fact, however, the luminosity density of the CfA1 sample appears to be $\sim 25\%$ higher than for the much larger (fair sample?) APM data (Tully, private communication). If the presence of rich structures like Virgo, Coma, and the Great Wall in this sample similarly indicates unusual higher power on larger scales, then the appropriate curve to compare to CfA1 may be closer to CHDM_1 than CHDM_2.

Figure 14 shows f_{gr} , the fraction of galaxies grouped, vs. V_5. Fraction grouped is a powerful statistic, since by separating our grouping algorithm into velocity and sky projected components, f_{gr} is sensitive not only to small scale pairwise velocity differences, but also the degree of spatial concentration of galaxies. CDM differs from CHDM significantly on both measures. As remarked earlier, the improved merging scheme applied to the CfA1 data here had the effect of performing fewer mergers. This left more CfA1 members in dense regions and thus raised the fraction of galaxies grouped; e.g. from 42% to 49% at the fiducial links. It had very little effect on the v_{gr} . Another change from NKP94 was to make CDM sky catalogs using only halos above $(\delta\rho/\rho)_{cut} = 156$ rather than 80. This had the effect of slightly raising CDM's v_{gr} 's by $\sim 3\%$. The net result is closer agreement between CHDM_2 and CfA1 f_{gr} curves. CHDM_1's higher power on large scales leads to a significantly higher correlation and a too high f_{gr} . CDM's high small scale pairwise velocities and puffier, less concentrated galaxy filaments inhibit grouping, especially near the fiducial links.

By combining Figures 13 and 14 to eliminate V_5, we can both enhance the differences between CDM and CHDM and produce a statistic which is very robust. This, our favored statistic, v_{gr} vs. f_{gr} , is shown in Figure 15. CHDM_2 to a slight extent, and CHDM_1 at the $\sim 2\sigma_{cv}$ level, groups too high a fraction of galaxies. CDM both groups too few galaxies and produces v_{gr} 's too high, at the $\sim 3\sigma_{cv}$ level for CDM1 and $\sim 4\sigma_{cv}$ level for CDM1.5. Notice that CDM1.5's curve actually lies above the CDM1 curve. The reason is that CDM1.5 groups a significantly lower fraction than CDM1. Thus it is more accurate to say that the CDM1.5 curve is *left* of the CDM1 curve. The suprisingly high discrimination shown by this statistic shows the power of grouping in redshift space. The "pressure" provided by higher small scale velocities will tend to expand structures on ~Mpc (i.e. galaxy group) length scales. This not only lowers the fraction of galaxies grouped, but also raises their rms velocities, and does so at all links. It is possible that f_{gr} may be sensitive to the presence of large clusters in relatively small samples like the CfA1 data. It is therefore important to see how v_{gr} vs. f_{gr} behaves on simulation data with the same sky coverage as CfA1, shown in Figure 15(b).

The local supercluster tends to raise $f_{\rm gr}$ for all simulations, but only slightly. Medians are insensitive to a few large groups, and median $v_{\rm gr}$'s show little change. The net conclusion remains the same, albeit with slighly higher sky variance. CHDM$_2$ again shows fair agreement with CfA1 for all V_5. Comparing data point by point shows that both CHDM's have $v_{\rm gr}$'s slightly too high (except for the last point). CHDM$_1$ now seems clearly to group too many galaxies. Figure 15 can be interpreted as indicating that group analysis favors a lower Ω_ν than the 0.30 used here.

The $f_{\rm gr}$ vs. $v_{\rm gr}$ statistic is also quite robust. Figure 16 shows $v_{\rm gr}$ vs. $f_{\rm gr}$ for the no breakup simulation sky catalogs, without $N(z)$ correction, for our three L(M) assignment methods. Using the (b) Cen-Ostriker L(M) prescription leaves the curves almost unchanged, even though the luminosity function's M^* is now ~ 2.5 magnitudes too bright and the faint end slope is $\alpha \sim -1.8$. The (c) Tully-Fisher (TF) prescription's L(M) differs even more drastically from observations, and is quite un-Schechter-like. Still, the resulting $v_{\rm gr}$ vs. $f_{\rm gr}$ curves are qualitatively similar to the Schechter and Cen-Ostriker results, and again CDM curves are too high while CHDM's curves are now too low. Note that all simulation TF $v_{\rm gr}$'s are lower. This is because the TF L(M) strongly picks out nearby, low-mass galaxies and groups and therefore gives lower median $v_{\rm gr}$'s. Comparing Figures 15(a) and 16(a) shows that breaking up the halos and correcting for differing redshift distribution lowers $v_{\rm gr}$, but still leaves CDM much too high and CHDM$_2$ in good agreement with observations.

Figure 17 compares the breakup methods on the $v_{\rm gr}$ vs. $f_{\rm gr}$ plane. Relatively little change is seen for any method when applied to CHDM, as shown in Figure 17(a) for CHDM$_2$. For CDM1, shown in Figure 17(b), all methods are close except for breakup Method 1. By forcing essentially equal, maximum possible masses for all DM halo fragments, Method 1 raises $f_{\rm gr}$ dramatically while lowering $v_{\rm gr}$. Figures 17(a) and 17(b) Method 1 curves lie virtually on top of one another. In fact, breakup Method 1 has the unfortunate property that all simulation curves are almost degenerate on this plane, and significantly below the CfA1 curve. While there is no reason to think that this breakup scheme is reasonable, it does show that it is possible to construct sky catalogs whose $v_{\rm gr}$ vs. $f_{\rm gr}$ properties are not discriminated.

Figures 18-20 show how $v_{\rm gr}$ vs. $f_{\rm gr}$ change when V_5 is held fixed and D_n is varied instead. Here, groups grow primarily on the sky and only secondarily in redshift depth. In NKP we described Figure 18, which does not have the $N(z)$ correction or break up included. Our conclusion there was that the dense cores picked out at small D_n were significantly "cooler" in CfA1 than for any simulation. Figure 19 shows that this is only true when overmergers are not broken up and no $N(z)$ correction is made for differing redshift distributions. When corrected in this way, CHDM at the fiducial V_5's actually reproduces the CfA1 results very well, while CDM $v_{\rm gr}$'s still remain too high. The largest effect is the $N(z)$ correction. As described earlier, the simulation groups tend to lie at higher distance, where the magnitude limit then identifies larger, higher $v_{\rm gr}$ groups. Comparing Figures 18 and 19 shows that $f_{\rm gr}$ in fact changed very little at these fiducial V_5's, while most of the change is in $v_{\rm gr}$. Figure 20 is similar to Figure 19, but for the 2.66 sr sky catalogs, and $v_{\rm gr}$'s are ~ 10 km s^{-1} higher.

Figures 21 and 22 shows the number of groups per steradian vs. redshift link, and vs. sky link, and demonstrate the percolation properties of the catalogs. As V_5 is raised, new groups will be identified, and some existing groups which are close in redshift will be merged. Beyond $V_5 = 350$ km s^{-1}, the CHDM$_1$ curve actually shows a significant decline as merging outpaces the production of new groups. An intermediate curve between the two CHDM curves would appear to fare much better with CfA1 groups, but appears likely to peak at both D_n and V_5 too low. Comparing such a CHDM curve to the CDM curves, which roughly peak together with CfA1, indicates that a lower Ω_ν would improve CHDM's agreement. Pure CDM, however, appears to significantly underproduce groups at small V_5. Thus, on this measure, CHDM appears to agree well if CfA1 has a moderately high amount of large scale power, and if we lower Ω_ν slightly.

10. Possible Problems

One quantity which shows significant disagreement with observations for all simulations is group size. Figure 23 shows the median r_h vs. D_n. All simulations are $50\% - 80\%$ higher than CfA1, at least near the fiducial link of $D_n = 0.36$. CDM actually fits slightly better, probably because CDM groups galaxies less efficiently and groups tend to be poorer and smaller than CfA1 groups. However, a more robust measure of group size, the mean pair separation r_p, shows simulation groups are only $\sim 20\%$ larger than CfA1 groups. This suggests that the problem is with residual limitations in our treatment of resolution, since r_h is very sensitive to the presence of close pairs. It may be that our enforcement of a 2-cell nearest neighbor limit on overmerger fragments was too severe and removed too many close pairs relative to CfA1, even after carefully merging CfA1. Or, it may instead reflect more fundamental limitations in our simulations. One possibility is that the overdense DM cells we call galaxies still retain some of the distribution properties of their parent DM, and are insufficiently "galaxy- like". Serna, *et al.* (1994) show that cluster galaxies residing in a DM background will show a stronger concentration than the DM, by a factor of about $r_h(DM)/r_h(gal) \simeq 2$, higher for older clusters. Hydrodynamic simulations (FEWS and references therein) also show baryonic galaxies are more concentrated within DM cluster potentials. If CfA1 galaxies are like Serna's galaxies, while our overdense DM cells behave somewhere between galaxies and DM particles, one might expect $r_h(gal)/r_h(DM)$ values similar to what we see. Finally, it may reflect a real disagreement between all of these models and reality.

Interestingly, our (disfavored) breakup Method 1 gave median r_h values which were closer to that for CfA1. This is because by imposing nearly equal masses on all fragments, a much larger fraction of fragments survived the magnitude cut. This produces many more small groups. Even so, the trend of median r_h with D_n was still steeply negative, similar to Figure 23, and in strong disagreement with CfA1. Also recall that Method 1 sky catalogs all produced virtually the same $v_{\rm gr}$ vs. $f_{\rm gr}$ curves and in poor agreement with CfA1.

11. Conclusions

Our first major conclusion is that these group statistics, especially $v_{\rm gr}$ vs. $f_{\rm gr}$, show strong sensitivity to the fraction of hot dark matter in CDM, while at the same time showing much less sensitivity to the normalization within CDM. The statistics appear quite robust to a variety of reasonable methods of illuminating the halos and breaking up overmergers, and to the relatively poor force resolution used here.

Our second conclusion is that COBE-normalized CHDM at $\Omega_\nu = 0.3$ produces group rms velocities quite similar to CfA1, but the fraction of galaxies grouped is about $2\sigma_{cv}$ too high, while CDM at b=1.0 and b=1.5 groups too few galaxies and gives $v_{\rm gr}$ too high, at the several σ_{cv} level. We now attempt to refine our estimate for an Ω_ν which is in optimal agreement with our group analysis.

The CHDM$_1$ power spectrum in real space $P_r(k)$ is a factor of 2 higher than that for CHDM$_2$ on scales comparable to the size of the box (KNP). The redshift space power $P_z(k)$ is amplified by a factor

$$P_z(k) = P_r(k)\big[1 + \frac{2}{3}\frac{\Omega^{0.6}}{b} + \frac{1}{5}(\frac{\Omega^{0.6}}{b})^2\big] \qquad (12)$$

(Kaiser 1987). For our $\Omega = 1$ b=1.5 CHDM simulations then, $P_z(k) = 1.5P_r(k)$, so that in redshift space CHDM$_1$ has $P_z(k)$ 3 times higher than that of CHDM$_2$. The CfA1 $P_z(k)$ is approximately a factor of 1.6 higher than that of the CfA2 at these scales (Vogeley, *et al.* 1992), while it may be more comparable to CfA2 on smaller scales. The CfA2 $P_z(k)$ is in turn a factor of 2 higher than that of the much larger APM Survey on similar scales (Baugh & Efstathiou 1993). While these scales are larger than typical group/clustering lengths, our CHDM_1 vs. CHDM_2 simulations show that higher power on these scales indeed "crosstalks" into higher fractions of galaxies grouped (see Figure 14), apparently aided by percolation along strong filaments. Such coupling of large to small scales has already been noted for pairwise velocities (Gelb, *et al.* 1993). Also, on galaxy scales the CfA1 galaxy luminosity density is a factor of 1.25 higher than that of the APM (Tully, private communication). If the APM can be taken as approaching a fair sample of the universe, the CfA1 then appears to have a redshift space power spectrum at large scales which is at least twice as high as is typical. This then suggests that the proper curve to compare to the CfA1 data is intermediate between those for CHDM_1 and CHDM_2, but closer to CHDM_1. Figures 14 and 15 then indicate an optimum Ω_ν somewhat lower than 0.3. While the detailed relation between Ω_ν and v_{gr} vs. f_{gr} is as yet unexplored, if it is approximately linear then these curves suggest an optimum $\Omega_\nu \simeq 0.2$. With the intermediate CHDM curve described above, Figures 21 and 22 would be in excellent agreement with the CfA1 data for any Ω_ν significantly less than 0.3. Figures 19 and 20 favor $\Omega_\nu \sim 0.3$, but are, within the errors, compatible with a slightly lower value. The total mass m_ν of all massive neutrino species is related to Ω_ν by $m_\nu = 23.51\Omega_\nu h_{50}^2$ eV, where h_{50} is the Hubble parameter in units of 50 km $\mathrm{s}^{-1}\mathrm{Mpc}^{-1}$, using the current CMB temperature of 2.726 K, and relevant parameters from Kolb & Turner (1990). The present group analysis' favored $\Omega_\nu \sim 0.20$ then corresponds to $m_\nu \simeq 4.6$ eV, for $H_0 = 50$ km $\mathrm{s}^{-1}\mathrm{Mpc}^{-1}$.

The void probability function is perhaps another indication that CHDM works better for lower Ω_ν (Ghigna *et al.* 1994). Also, the abundance of massive objects is very sensitive to the fraction of hot dark matter, as these objects are far out on the exponential tail of the mass distribution at high redshift, especially for cosmologies which form structure late, such as CHDM. Recent observations now suggest that an Ω_ν larger than ~ 0.25 (Klypin *et al.* 1995a) is incompatible with the abundance of damped Lyα systems at redshifts of $z \sim 3 - 4$ (Storrie-Lombardi 1995), although this limit depends somewhat on the still poorly known baryon fraction Ω_b

The third major conclusion is that, within the models studied here, it is quite natural to find $\Omega \sim 0.1$ from group mass to light ratios, even though the true Ω is 1. Three factors combine in the same direction to severely bias the M/L method on the low side. First, galaxies within a group occupy only the central core of much larger DM concentrations. The total mass bound to the group at $2r_h$ is a factor of 3 higher than the DM within r_h. Second, the appropriate virial mass to calculate is that due to the individual cold DM particles, not the galaxies, whose velocity bias contributes another factor of ~ 2. Finally, the virial theorem implicitly assumes spherical symmetry, and our groups are actually fairly elongated. Counting mass within these elongated boundaries will add perhaps another 30% to the true mass, giving a net correction to M/L of nearly an order of magnitude. Hot DM which is unclustered will further bias M/L to the low side.

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Figure Captions

Figure 1. Comparison of the two methods of assigning velocities to the fragments of (a) CHDM and (b) CDM1 overmerger breakups. V^{om}_{neigh}, the rms velocity of all neighboring galaxies within 1 Mpc , or within whatever larger radius encloses 4 neighbors, has a much wider range than the characteristic circular velocity $V_c^{om} = (GM_{eff}/r_{eff})^{1/2}$, yet the medians are very similar. CDM1 has a wider range for both velocities than does CHDM.

Figure 2. Comparison of the halo 1-cell mass distribution for the different massive halo overmerger breakup methods. Our favored method is labeled "breakup", while Methods 1 and 2 are labelled "breakup$_1$" and "breakup$_2$". d$LogN$/d$logM$ $= -1.3$ below the breakup mass M_{bu}.

Figure 3. The median group rms velocity vs. fraction grouped for the full boxes cut at $\delta\rho/\rho = 80$. From left to right, the points on each curve correspond to D_0=0.10, 0.15, 0.20, 0.25, 0.30, 0.36, 0.47, 0.58, 0.72. D_0=0.10 gave no groups for CDM1 and CDM1.5 and this point is skipped. At low $f_{\rm gr}$ only the dense cores of richer groups are selected, giving higher $v_{\rm gr}$ (but small sample size makes these points more uncertain).

Figure 4. The DM density distribution for (a) the CHDM box groups with halos of $\delta\rho/\rho > 80$, and (b) the CDM1 and CDM1.5 box groups with halos of $\delta\rho/\rho > 156$, compared to that for r^{-2} isothermal spheres. r=0 is the uweighted average of the position vectors of all member halos. Only the curve for total mass is shown for CHDM$_1$, while CHDM$_2$ shows the hot and cold distributions as well. Even at $r = 2r_h$, mass is substantially above the critical density ρ_c. All simulations show an exponential $\rho(r)$, but CDM shows a steeper fall-off than CHDM. Data inside $\sim$ 195 kpc is unresolved and not plotted.

Figure 5. The cumulative mass distribution around (a) the fiducial box groups and (b) the fiducial sky groups. There is substantial mass outside but bound to the box groups; $M(2_rh)/M(r_h)$ is 2.7 for CHDM and nearly 2.0 for CDM. Around the sky groups, $M(2_rh)/M(r_h)$ is as high as 3.0 for all simulations. Blurring in redshift space reduces the differences between CHDM and CDM curves in the sky groups. Data inside $\sim$ 195 kpc is unresolved and not plotted.

Figure 6. The ratio of cold to hot dark matter for the CHDM box groups (unweighted average over all groups). It is more representative of small groups, which make up most of this volume-limited sample. Group cores are relatively dense and cold, with the hot particles predominately forming a more diffuse background.

Figure 7. (a) The assigned 1-cell mass-to-light ratio required in order to reproduce the CfA1 Schechter luminosity function parameters, for each simulation. There is only a single curve for the Cen-Ostriker and Tully-Fisher prescriptions, since no attempt is made to force the resulting luminosity function to agree with observations. The steep rise in M/L at low mass means low mass galaxies do not appear in the sky catalogs. (b) The distribution of no breakup sky catalog galaxies vs. mass. (c) Breaking up overmergers clips the high end and makes a large number of fragments below M_{bu}. Tully-Fisher (not shown) and Cen-Ostriker L(M)'s put much higher luminosity on faint, low mass, nearby galaxies.

Figure 8. The median axial ratio (depth divided by average of RA and Dec dimensions) in real space of redshift-selected groups, vs. V_5. The CHDM curves flatten at low V_5, indicating minimal contamination by interlopers here. CDM curves are steeper and more contaminated. For comparison, Poisson distributed galaxies of rms peculiar velocities $\sigma = 200$ and 350 km s^{-1} are much steeper still, showing even CDM galaxies have significant coherent motion.

Figure 9. The number-weighted virial mass estimator M_{vir} from the halos, vs. the dark matter M_{DM} contained within the mean harmonic radius r_h defined by the halos, for 3D selected box groups in CDM1. The no-breakup case (a) has a significant tail of high M_{vir}/M_{DM} groups, while the breakup case (b) includes many more $M_{vir}/M_{DM} \sim 1$ groups. Higher pairwise velocities in CDM cause the extended tail to high M_{vir}/M_{DM} in both cases. The combined sky groups for all six viewpoints (c) show a strong bias towards high M_{vir}/M_{DM} at low mass, as high pairwise velocities mean small sky groups are not bound.

Figure 10. The same as Figure 9, but for CHDM$_2$. There is little difference between the (a) no-breakup and (b) breakup cases, and the low pairwise velocities make for less bias and less noise in the (c) sky groups' M_{vir} than seen in CDM.

Figure 11. The velocity bias parameter b_v for all box groups at the fiducial link. The horizontal lines define the median values. CHDM$_2$ has a significant bias of $b_v = 0.7$. CDM1 shows almost no net velocity bias. CHDM$_2$'s low pairwise velocities lead to a tighter distribution. The banding in CDM1.5 (d) is due to the small subsample of dark particles used in the calculations.

Figure 12. (a) Galaxy density vs. redshift in the magnitude limited sky catalogs for CHDM$_2$ vs. that for CfA1. The simulations' densities drop less steeply with redshift due to the limited box size (and cosmic variance). Taking random sub-samples of simulation groups which match CfA1 groups' distribution in z (the $N(z)$ correction) insures that median properties have no distance-dependent bias. (b) $v_{\rm gr}$ vs. redshift. $v_{\rm gr}$ typically rises with distance as only the richer, more luminous groups survive the magnitude limit. CfA1 groups are relatively overabundant nearby and scarce beyond $\sim$ 8000 km s^{-1}. The number of groups per unit redshift is roughly constant from $V \sim 1000 - 7000$ km s^{-1} as the rising volume sampled competes with the increasingly severe luminosity cut.

Figure 13. Median $v_{\rm gr}$ vs. V_5, for sky catalogs from the breakup boxes and the CfA1. CHDM curves match the observations well, while CDM's are too high. Below $\sim$ 400 km s^{-1}, curves are degenerate since the CDM groups are in the "clipped" regime. On this and later curves, error bars are 1σ over the 6 viewpoints.

Figure 14. $f_{\rm gr}$ vs. V_5 for all simulations. CDM groups too few galaxies, while CHDM groups slightly too many. CHDM$_1$'s higher power on large scales apparently aids the formation of groups.

Figure 15. Our favored statistic $v_{\rm gr}$ vs. $f_{\rm gr}$ under varying V_5 for our fully corrected case using (a) the 10.384 sr sky catalogs and (b) the 2.66 sr CfA-sky catalogs. In the $f_{\rm gr}$ direction, CHDM curves are 1-2 σ_{cv} high, while CDM curves are at least $3\sigma_{cv}$ too low, assuming the difference between CHDM$_1$ and CHDM$_2$ is a $1.7\sigma_{cv}$ estimate of cosmic variance.

Figure 16. The robustness of $v_{\rm gr}$ vs. $f_{\rm gr}$ against luminosity assignment methods; (a) Schechter, (b) Cen-Ostriker, and (c) Tully Fisher prescriptions, all done before breaking up overmergers or applying an $N(z)$

correction. In all cases, CDM is substantially too high. The TF case is probably the least reliable method, as its luminosity function gives a very poor fit to the Schechter form. All methods give similar conclusions, though TF predicts a lower Ω_ν than the others.

Figure 17. Comparison of $v_{\rm gr}$ vs. $f_{\rm gr}$ for different overmerger breakup methods for (a) $\rm CHDM_2$ and (b) CDM1. Other simulations are similar. All methods except Method 1 lead to CDM curves in conflict with observations by several σ_{cv} Method 1 collapses all simulations onto the same (too low) curve (e.g. (a) and (b) dotted curves virtually overlap) by forcing all fragments to have the maximum possible, nearly equal brightness.

Figure 18. Median $v_{\rm gr}$ vs. $f_{\rm gr}$ when varying D_n and holding V_5 constant at the two different fiducial values (a) $V_5 = 600$ and (b) $V_5 = 350$ km s^{-1}, both with no breakup of overmergers or $N(z)$ correction. At $V_5 = 350$ the CDM groups are in the "clipped" regime. A better comparison is at $V_5 = 600$ km s^{-1}. NKP94 used this figure to prematurely claim CfA1 groups had significantly cooler cores than any simulations.

Figure 19. The same as Figure 18, but using the $N(z)$ corrected breakup catalogs The "cooler cores" conclusion of NKP94 is now seen to be an artifact of overmerging and $N(z)$ bias. When corrected, CHDM is in reasonable agreement with observations at both fiducial V_5's, while CDM remains too high.

Figure 20. The same as Figure 19, but now using the 2.66 sr sky catalogs. The error bars are larger, but otherwise the figure is virtually identical to that for the full 10.384 sr catalogs. At low and moderate V_5, $f_{\rm gr}$ differs very little between these different sky cuts.

Figure 21. Percolation properties of groups vs. varying V_5 in the fully corrected case for the CfA-sky catalogs. CDM produces too few groups at low V_5, while $\rm CHDM_2$ produces too many at intermediate and high V_5. $\rm CHDM_1$ percolates too easily, as groups merge faster than they are created above $V_5 = 450$ km s^{-1}. A curve intermediate between $\rm CHDM_1$ and $\rm CHDM_2$ would likely give the best fit, though discrimination between models is poor in this plane.

Figure 22. Percolation properties of groups vs. varying D_n for all simulations, using the 2.66 sr sky catalogs. As in Figure 21, $\rm CHDM_1$ percolates too easily. $\rm CHDM_2$ percolates correctly, but produces too many groups at all D_n. A high large scale power CHDM model with lower Ω_ν would appear to fit better. Both CDM models fit well on this measure.

Figure 23. The ratio of the simulation mean harmonic radius r_h to that for the CfA1 at the same links, vs. D_n. All simulations are significantly too high. This may be due to residual resolution limitations, or to simulation halos having spatial distributions not sufficiently galaxy-like.